CHAPTER

14

Competitive Market Equilibrium

Until now, we have only proceeded through the first two of three steps of the "economic way of thinking" — the crafting of a model and the process of optimizing within that model. We now proceed to the final step: to illustrate how an equilibrium emerges from the optimizing behavior of individuals — how the "economic environment" that individuals in a competitive setting take as given emerges from their actions. In the process, we begin to get a sense of how order can emerge "spontaneously" — an idea introduced briefly in the introduction as one of the big ideas that we should not loose as we dive into technical details of economic models.

Chapter Highlights

The main points of the chapter are:

1. An **equilibrium** arises in an economic model when no individual has an incentive to change behavior given what everyone else is doing. In a competitive model, it means that no individual has an incentive to change behavior given the economic environment that has emerged "spontaneously".

2. The **short run for an industry** is the time over which the number of firms in the industry is fixed because firms have not had an opportunity to enter or exit the industry. The **short run industry (or market) supply curve** is therefore the sum of the firm supply curves for the (short run) fixed number of firms in the industry, and the **short run equilibrium** is driven by the price at which the short run industry supply curve intersect the market demand curve (which is simply the sum of all individual demand curves).

3. The **long run for an industry** is the time it takes for sufficient numbers of firms to enter or exit the industry as conditions change. The **long run industry (or market) supply curve** therefore arises from the condition that the *marginal* firm in the industry must make zero profits so that no firm in the industry has an incentive to exit and no firm outside the industry has an incentive to enter. When all firms face identical costs, this implies a horizontal

long run industry supply curve at the price which falls at the lowest point of each firm's long run *AC* curve. The **long run equilibrium** then emerges at the intersection of market demand and long run industry supply.

4. To analyze what happens as conditions change in a competitive market, the most important curves to keep track of on the firm side are the (1) **long run *AC* curve** and (2) the **short run firm supply curve that crosses the long run *AC* curve at its lowest point** (but extends below it because shut down prices are lower than exit prices.) Any change that impacts short run firm supply curves will impact the short run industry supply curve, and any change that impacts the long run *AC* curve will impact the long run industry supply curve.

5. Changes that affect only a single firm in an industry do not affect the market equilibrium in either the short or the long run.

Using the LiveGraphs

For an overview of what is contained on the LiveGraphs site for each of the chapters (from Chapter 2 through 29) and how you might utilize this resource, see pages 2-3 of Chapter 1 of this *Study Guide*. To access the LiveGraphs for Chapter 14, click the *Chapter 14* tab on the left side of the LiveGraphs web site.

We do not at this point have additional *Exploring Relationship* modules for this chapter.

14A Solutions to Within-Chapter-Exercises for Part A

Exercise 14A.1 *Can you explain why there is always a natural tendency for wage to move toward the equilibrium wage if all individuals try to do the best they can?*

Answer: If wage were to drift below the equilibrium, firms would not be able to fill all their job vacancies because not enough workers are willing to work at a below-market wage. Thus, it is in each firm's interest to offer a slightly higher wage in order to fill its positions — and this continues to be true until all positions are filled at the equilibrium wage. If, on the other hand, the wage were to drift above the equilibrium, some workers who want to work will be unable to find a position. It would be in their interest to offer to work for slightly less so that they can get employed when there are fewer jobs than workers wishing to take them — and this continues until the wage falls at the equilibrium where the number of jobs is exactly equal to the number of workers willing to take them.

Exercise 14A.2 *Suppose your firm only used labor inputs (and not capital) and that labor is always a variable input. If your firm had to renew an annual license fee, would the AC^{SR} and the long-run AC curves ever cross in this case?*

Answer: No, in this case the only difference between AC and AC^{SR} is the cost of the license fee — which does not vary with output. Thus, AC lies above AC^{SR} but converges to it as x increases.

Exercise 14A.3 *Why might the AC^{SR} and the long-run AC curves cross when the difference emerges because of an input (like capital) that is fixed in the short run? (*Hint*: Review Graphs 13.2 and 13.3.)*

Answer: This is because the fixed *expense* associate with the input that is fixed in the short run becomes a *variable* cost in the long run. It is therefore different than a license fee that does not change with the level of output — the cost associated with the input increases as output increases. Suppose, for instance, that capital is the fixed input in the short run and therefore is not part of short run average cost. For high levels of output, the fixed capital level may be sufficiently low such that very high levels of labor are necessary to make up for it. This may cause the short run labor costs to exceed the long run costs of both labor and capital if the firm in the long run can substitute a lot of the labor for relatively little capital.

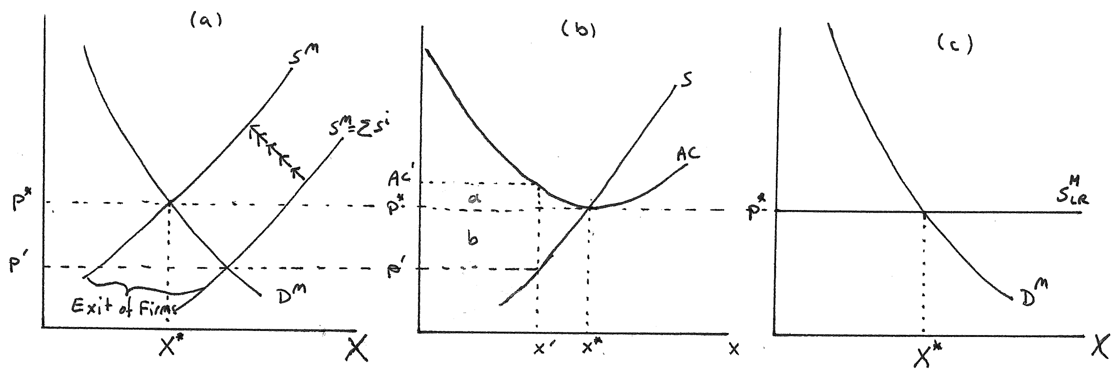
Exercise 14A.4 *Explain why the MC curve in Graph 14.4 would be the same in the long and short run in the scenario of exercise 14A.2 but not in the scenario of exercise 14A.3.*

Answer: This is because in the case of a fixed cost (like a license fee), the cost does not change with output — which implies the MC curve does not change even if the AC curve shifts up. But if the fixed expense in the short run becomes a variable cost in the long run (as happens with a fixed input that becomes variable), then the MC curve changes because the cost associated with the input changes with output as the input becomes variable in the long run.

Exercise 14A.5 *Can you draw the analogous sequence of graphs for the case when the short run equilibrium price falls below p^*?*

Answer: This is illustrated in Graph 14.1 (next page). The long run equilibrium price p^* falls at the lowest point of (long run) AC for the firms (where profit is zero). If the short run equilibrium price p' falls below p^* as drawn in panel (a), each firm produces x' along its short run supply curve as illustrated in panel (b). This implies that the average cost AC' is higher than p', leading to long run profit that is negative and equal to $(-a-b)$ in the graph. As a result, firms will exit — shifting the short run market supply curve in panel (a) to the left until price reaches p^*.

Exercise 14A.6 *How does the full picture of equilibrium in Graph 14.2 look different in the long run?*



Graph 14.1: Movement to Long Run Equilibrium when price is below *AC*

Answer: The long run price would settle at the lowest point of the (long run) *AC* curve of firms. Thus, panel (f) would only need to show the long run *AC* curve, and the intersection of D^M and S^M in panel (e) would occur at the price equal to the lowest level of the *AC* curves of firms.

Exercise 14A.7 *How would you think the time-lag between short and long run changes in labor markets is related to the "barriers to entry" that workers face, where the barrier to entry into the PhD economist market, for instance, lies in the cost of obtaining a PhD.*

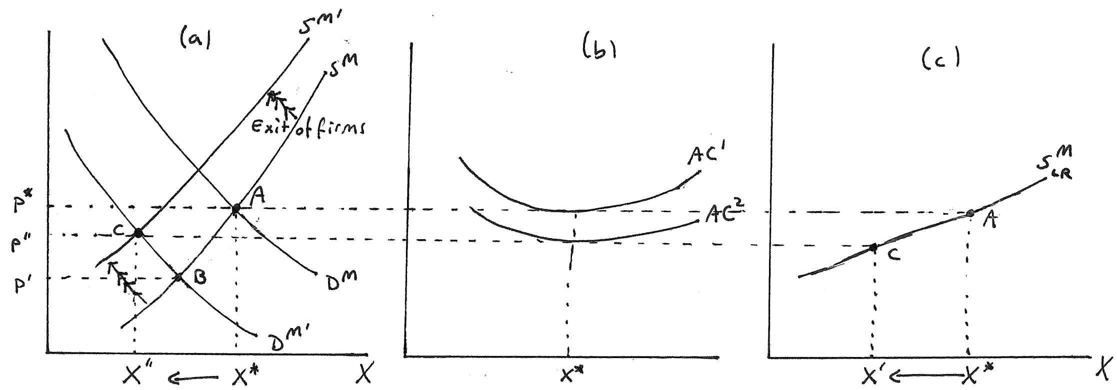
Answer: The greater the barriers to entry, the longer it will take for the labor market to reach the long run equilibrium.

Exercise 14A.8 *Can you explain why the previous sentence is true?*

Answer: As long as the lowest points of the *AC* curves of firms are all at the same vertical height in the graph, all firms have the same exit (or entry) price — and thus exit and entry decisions by firms will drive long run price to that level. It does not matter whether these lowest points of *AC* curves occur at different output levels for different firms — i.e. whether these lowest points are horizontally different. If they are, it simply implies that different firms will produce different levels of output in long run equilibrium, but their exit/entry prices are all the same (so long as the *AC* curves do not differ vertically.)

Exercise 14A.9 *Suppose market demand shifts inward instead of outward. Can you illustrate what would happen in graphs similar to those of Graph14.6?*

Answer: Graph 14.2 (next page) illustrates this. In panel (a), D^M and S^M intersect at the original equilibrium A — with price at p^* and industry output at X^*. At that price, any firm with AC at or below p^* is producing x^* — as, for instance, both the firms in panel (b). (This is assuming the lowest point of all AC curves occurs at the same output quantity). When demand shifts to $D^{M'}$ as illustrated in panel (a), the initial short run equilibrium shifts to B. Since the marginal firms were making zero profit before, they are now making negative long run profit — implying that they will begin to exit, which in turn causes the short run market supply curve in panel (a) to shift to the left. This continues to happen until the marginal firm left in the industry makes zero profit. This is illustrated as firm 2 with average cost curve AC^2 in panel (b). Since the highest cost firms exit, the new equilibrium price p'' will fall below p^* — leading to the upward sloping long run market supply curve in panel (c).



Graph 14.2: Inward shift in D^M

Exercise 14A.10 True or False*: The entry and exit of firms in the long run insures that the long run market supply curve is always shallower than the short run market supply curve.*

Answer: This is true. One way to see this is to think about changes in demand for an industry that is initially in long run equilibrium (before the change in demand). Suppose demand increases. This implies that industry output will rise as each firm produces more at the higher price (that results from the new intersection of (short run) market supply and demand. But, since we started in long run equilibrium, all firms that were initially making zero profit must now be making higher profit — which gives an incentive to firms outside the industry to *enter*. This will shift the short run market supply curves, driving down the price and increasing industry output until the marginal firm makes zero profit again. Thus, the entry of new firms causes the long run output increase to be larger than the short run increase. The reverse happens when demand falls. In that case, firms will produce less as price falls to the new intersection of demand and short run market supply. But since they were initially making zero profit, this implies they will not make negative (long run) profit — which implies some of the firms will exit the industry. This will shift the market supply curves inward, raising price back to the lowest point of the firms' (long run) *AC* curves. That shift then causes a further decrease in industry output. Thus, whether demand increases or decreases, the long run response is larger than the short run response — meaning that long run industry supply curves are shallower than short run industry supply curves because of entry and exit of firms in the long run.

Exercise 14A.11 True or False*: While long run industry supply curves slope up (in increasing cost industries) because firms have different cost curves, long run industry supply curves in decreasing cost industries slope down even if firms have identical cost curves.*

Answer: This is true. The text demonstrates how upward sloping long run industry supply curves arise from firms having different cost curves — with higher cost firms entering industries as industries expand — and thus price increasing to insure zero profit for marginal firms. In decreasing cost industries, however, the downward slope of industry supply curves is not due to different costs for firms — and in fact, if firms did have different cost curves, it would be much more likely that an industry could ever have a downward sloping long run supply curve. Rather, the downward sloping industry supply curve arises because *all* firms experience lower costs as the industry expands — i.e. firm costs are changing as the industry expands and input prices fall.

Exercise 14A.12 True or False*: In the presence of fixed costs (or fixed expenditures), short-run profit is always greater than zero in long run equilibrium.*

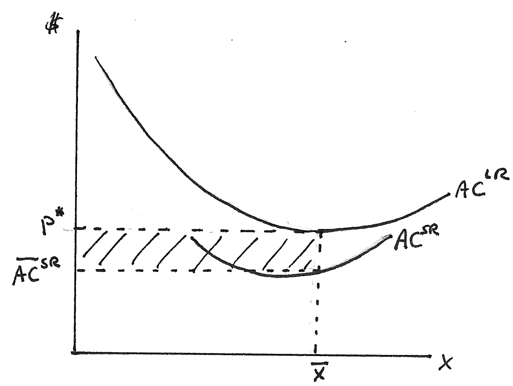
Answer: This is true. This is because short run profits do not include these fixed (long-run) costs while long run profits do.

Exercise 14A.13 *Can you illustrate graphically the short and long run profits of the marginal firm in long run equilibrium? (*Hint*: You can do this by inserting into the graph the AC^{SR} curve as previously pictured in Graph 14.4.)*

Answer: This is illustrated in Graph 14.3 (next page) where the equilibrium price p^* causes the marginal firm to produce $\overline{x}$ at the lowest point of its long run average cost curve AC^{LR}. This implies short run average costs of AC^{SR}. From the short run perspective, the firm therefore incurs costs of $\overline{AC}^{SR} * \overline{x}$ but earns revenues of $p^* * \overline{x}$. The difference between these — indicated by the shaded area in the graph — is the short run profit earned by the marginal firm in long run equilibrium.

Exercise 14A.14 *Why does the increase in the fee result in a new (green) AC' curve that converges to the original (blue) AC curve?*

Answer: This is because the increase in the fixed fee does not depend on the level of output — so, on average, the additional fee becomes less as output in-



Graph 14.3: Short Run Profit in Long Run Equilibrium

creases. Put differently, if the increased fixed fee is F, the average increased fixed fee is F/x — which is F when $x = 1$ but converges to zero as x gets large.

Exercise 14A.15 *If you add the firm's long-run supply curve into panel (b) of the graph, where would it intersect the two average cost curves? Would the same be true for the firm's initial short-run supply curve? (*Hint*: For the second question, keep in mind that the firm will change its level of capital as its output increases.)*

Answer: The firm's long run supply curve would intersect the two long run AC curves at their lowest points (because the long run supply curve for a firm is the portion of its long run MC curve that lies above its long run AC curve.) In fact, it would initially begin at the lowest point of the blue AC curve — and would get shorter as a result of the increase in the recurring fixed cost, starting at the lowest point of the green AC' curve after the increase in the recurring fixed cost.

The firm's initial short run supply curve would also cross the lowest point of the initial blue long run AC curve (since the industry is initially in long run equilibrium). But it would (almost certainly) not cross the lowest point of the green AC' curve. This is because the level of capital that the firm has in the initial long run equilibrium is unlikely to be the level of capital it will end up with in the new long run equilibrium — and its short run supply curve therefore takes a (long-run) sup-optimal level of capital as fixed.

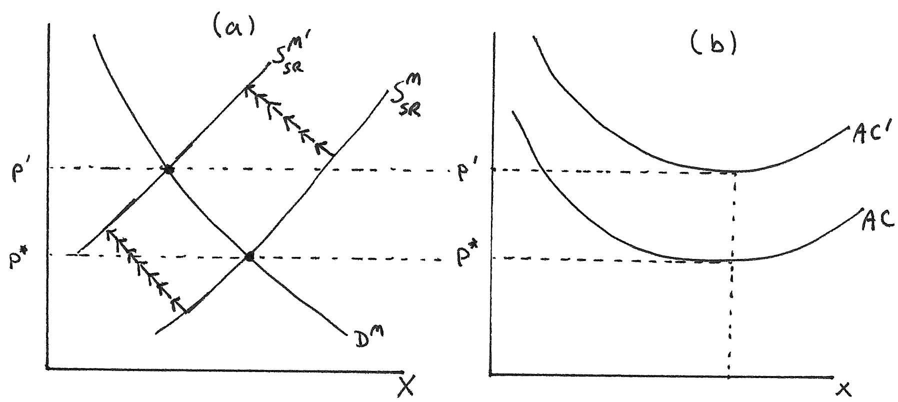
Exercise 14A.16 *Could the AC curve shift similarly in the case where the increase in cost was that of a long run fixed cost?*

Answer: No, it could not. In the case of a long run fixed cost, the (long run) MC curve remains unchanged because the fixed cost does not change the *additional* cost of producing any of the units of output. The MC curve also has to cross the AC

curve both before and after the increase in the fixed cost — which means that the lowest point of the *AC* curve must shift to the right when the fixed cost increases.

Exercise 14A.17 *How would you illustrate the transition from the short run to the long run using graphs similar to those in panels (a) and (b) in Graph 14.8?*

Answer: When long run costs increase, firms exit the industry. This shifts the short run supply curve to the left — driving price up until it reaches the lowest point of the firms' *AC* curves. This is illustrated in Graph 14.4.



Graph 14.4: Transitioning to Long Run Equilibrium

Exercise 14A.18 *Consider two scenarios: In both scenarios, the cost of capital increases, causing the long run AC curve to shift up, with the lowest point of the AC curve shifting up by the same amount in each scenario. But in Scenario 1, the lowest point of the AC curve shifts to the right while in Scenario 2 it shifts to the left. Will the long run equilibrium price be different in the two scenarios? What about the long run equilibrium number of firms in the industry?*

Answer: The only thing that matters for where the long run equilibrium price will settle is the vertical height of the lowest point of the long run *AC* curves of firms. Since this is the same in both scenarios, the long run equilibrium price will be the same for both cases. Thus, the long run market supply curve will intersect the market demand curve at the same point — which implies industry output will also be the same in both scenarios. But each firm will *increase* its production in the new equilibrium in Scenario 1 while each firm will *decrease* its production in the new equilibrium in Scenario 2. Thus, in order for the industry to produce the same in both scenarios, it must be the case that the equilibrium number of firms will be larger in Scenario 2 than in Scenario 1.

Exercise 14A.19 *How can it be that firms are making short run profit (and thus remain open in the short run) while simultaneously making negative long run profit (causing some of them to exit and thus price to rise further)?*

Answer: This is because expense associated with the capital that is fixed in the short run is not a short run cost — and therefore not subtracted when we calculate short run profit. It is, however, a long run cost — and therefore does get subtracted from long run profit.

Exercise 14A.20 *What would happen if instead the government imposed a per unit tax for each packet of economist cards?*

Answer: This would increase the *MC* of each unit of output by exactly the same amount. Thus, the *MC* curve shifts up by the same amount everywhere, shifting the short run market supply curve and causing price to rise and industry output to fall in the short run. In the long run, if each unit of output is taxed by the same amount, the long run *AC* curve shifts up by the same amount everywhere. Thus, the long run equilibrium price rises. Since *AC* shifts by the same amount for all output levels, its lowest point must remain at the same output level as before. This implies that, although firms reduce output in the short run, their output increases back to the original level in the long run. With the industry producing less at the higher price, this implies that some firms must have exited (because of negative long run profits otherwise), driving price up by more than in the short run. The number of firms therefore declines, but each firm (that remains in the industry) produces the same as originally (but at a higher price).

Exercise 14A.21 True or False*: Regardless of what cost it is, if it increases for only one firm in a competitive industry, that firm will exit in the long run but it might not shut down in the short run.*

Answer: This is true. Before the increase in costs, this firm was making zero long run profit. The increase in its costs does not change the equilibrium price since the firm is "small" — and thus long run profit is negative after the increase in the cost. This implies the firm will exit in the long run. In the short run, the firm continues to produce if the cost that increases is one associated with a short-run fixed input or a long run fixed cost — because these increases do not affect short run economic costs. If the increase in costs is an increase in a true short run cost (such as the cost of labor), then the firm's short run *MC* curve shifts to the left, causing it to either produce less (if short run profit does not become negative) or to shut down (if short run profit has become negative.)

Exercise 14A.22 *Replicate Table 14.1 for the cases where the demand and the various cost examples decrease rather than increase.*

Answer: This is done in the table that follows.

Example	Affected Costs SR	LR	Market Price	Industry Output	Firm Output	LR # of Firms
$\downarrow$ License Fee	None	AC	$\_^{SR} \downarrow^{LR}$	$\_^{SR} \uparrow^{LR}$	$\_^{SR} \downarrow^{LR}$	$\uparrow$
$\downarrow r$	None	AC, MC	$\_^{SR} \downarrow^{LR}$	$\_^{SR} \uparrow^{LR}$	$\_^{SR} ?^{LR}$	?
$\downarrow w$	AC, MC	AC, MC	$\downarrow^{SR} \downarrow^{LR}$	$\uparrow^{SR} \Uparrow^{LR}$	$\uparrow^{SR} ?^{LR}$	?
$\downarrow$ Demand	None	None	$\downarrow^{SR} \_^{LR}$	$\downarrow^{SR} \Downarrow^{LR}$	$\downarrow^{SR} \_^{LR}$	$\downarrow$

14B Solutions to Within-Chapter-Exercises for Part B

Exercise 14B.1 *Why is the demand function not a function of income?*

Answer: This is because the utility function from which it was derived is quasi-linear — which removes income effects from demand.

Exercise 14B.2 *Demonstrate that the average cost of production is U-shaped and reaches its lowest point at x =1280 where AC=5. (*Hint*: You can illustrate the U-shape by showing that the derivative of AC is zero at 1280, negative for output less than 1280 and positive for output greater than 1280.)*

Answer: Taking the derivative of $AC(x)$, we get

$$\frac{\partial AC(x)}{\partial x} = \frac{0.167185}{x^{3/4}} - \frac{1280}{x^2}. \tag{14.1}$$

Substituting $x = 1280$, we get $AC(1280) = 0$, and, for $x < 1280$, the function is indeed negative while for $x > 1280$ it is positive. We therefore have a U-shaped AC curve that reaches its lowest point at $x = 1280$. At that output level, the average cost is

$$AC(1280) = 0.66874(1280)^{1/4} + \frac{1280}{1280} = 5. \tag{14.2}$$

Exercise 14B.3 *Verify these individual production and consumption quantities.*

Answer: Substituting $p = 5$ into the consumer demand equation, we get

$$x^d(5) = \frac{625}{5^2} = 25, \tag{14.3}$$

and substituting $p = 5$ into the firm supply equation $x^s(p)$, we get

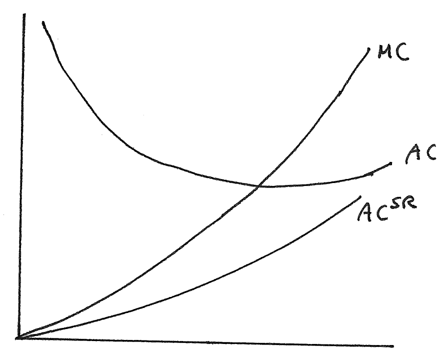
$$x^s(5) = 437.754(5)^{2/3} = 1280. \tag{14.4}$$

Exercise 14B.4 *We have already indicated that* $k = 256$ *is in fact the optimal long run quantity of capital when* (p, w, r)=(5,20,10). *Can you then conclude that the industry is in long run equilibrium from the information in the previous paragraph? (*Hint*: This can only be true if no firm has an incentive to enter or exit the industry.)*

Answer: Since we know $k = 256$ is the long run optimal quantity of capital, the short run expenditures of \$3,840 become economic costs in the long run. (If we did not know $k = 256$ was long run optimal, we would not be able to conclude this since the firm would further adjust capital and therefore the long run cost of capital would differ from the short run expense on capital.) Thus, from a long run perspective, the firm has \$3,840 more in costs than it has in the short run. Since we concluded that short run profit is \$3,840, this implies long run profit is \$0. The industry is therefore in long run equilibrium — with no firm in the industry wanting to exit and no firm outside the industry wanting to enter.

Exercise 14B.5 *Can you graph the* AC^{SR} *into panel (c) of Graph 14.12?*

Answer: This is illustrated in Graph 14.5 where the short run average cost curve must lie below the short run MC curve.



Graph 14.5: AC^{SR} with Decreasing Returns to Scale Production

Exercise 14B.6 *Why does the long-run profit become negative \$960 if nothing changes?*

Answer: Since we started in long run equilibrium, we know that initially the firms were making zero long run profit. When the license fees are increased by \$960, this then implies that long run profit must fall to minus \$960 if nothing changes.

Exercise 14B.7 *Verify these calculations.*

Answer: The lowest point of the AC' curve occurs where its derivative is zero — i.e. where

$$\frac{\partial AC'(x)}{\partial x} = \frac{0.167185}{x^{3/4}} - \frac{2240}{x^2} = 0 \tag{14.5}$$

which solves to $x = 2002.8$ or approximately $x = 2000$. At $x = 2000$, the average cost is

$$AC'(2000) = 0.66874(2000)^{1/4} + \frac{2240}{2000} \approx 5.59. \tag{14.6}$$

At that price, the market demand curve tells us that the consumers' demand is $D^M(5.59) = 40,000,000/(5.59^2) \approx 1,280,000$, with each individual consumer demanding $x^d(5.59) = 625/(5.59^2) \approx 20$. Each firm will supply about 2000 units of output — and with a total of about 1,280,000 produced by the industry, this implies that the new number of firms in the industry will be 1,280,000/2000≈ 640.

Exercise 14B.8 *Compare the changes set off by an increase in the license fee to those predicted in Graph 14.8.*

Answer: The graph suggests that we will see an increase in the quantity supplied by each firm with a decrease in quantity supplied by the industry at a higher price. Here we have calculated price increasing from \$5 to \$5.59, the industry quantity falling from 1,600,000 to 1,280,000 and and the amount produced by each firm *increasing* from 1,280 to 2,000. This is consistent with the directions of changes identified in the graph.

Exercise 14B.9 *Verify these calculations.*

Answer: The lowest point of the AC' curve occurs where its derivative is zero — i.e. where

$$\frac{\partial AC'(x)}{\partial x} = \frac{0.204759}{x^{3/4}} - \frac{1280}{x^2} = 0 \tag{14.7}$$

which solves to $x = 1088.36$ or approximately $x = 1088$. At $x = 1088$, the average cost is

$$AC'(1088) = 0.819036(1088)^{1/4} + \frac{1280}{1088} \approx 5.88. \tag{14.8}$$

At that price, the market demand curve tells us that the consumers' demand is $D^M(5.88) = 40,000,000/(5.88^2) \approx 1,156,925$, with each individual consumer demanding $x^d(5.88) = 625/(5.88^2) \approx 18$. Each firm will supply 1088 units of output, which implies that the total number of firms will be 1,156,925/1088≈ 1063.

Exercise 14B.10 *Are these results consistent with Graph 14.9?*

Answer: The graph suggests that industry output will fall while firm output remains unchanged and price increases. We calculated that industry output falls from 1,600,000 to 1,156,925 and price rises from \$5 to \$5.88. These results are consistent with the graph. We also calculated that each firm's output will fall from 1,250 to 1,088 which is different from what is shown in the graph. However, in developing the graph, we noted that the lowest point of the long run *AC* curve might shift to the left or right as the rental rate of capital increases — and we just happened to draw it as shifting in neither direction. So, while the mathematical results in this example are not consistent with how we drew the graph, they are consistent with our discussion in part A of the chapter.

Exercise 14B.11 *How much capital and labor are hired in the industry before and after the increase in r?*

Answer: In Chapters 12 and 13, we calculated the input demand functions for this technolgoy to be

$$\ell(p,w,r) = 32768\frac{p^5}{r^2 w^3} \quad \text{and} \quad k(p,w,r) = 32768\frac{p^5}{w^2 r^3}. \tag{14.9}$$

When $p = 5$ and $w = 20$, these become

$$\ell(r) = \frac{12800}{r^2} \quad \text{and} \quad k(r) = \frac{256000}{r^3}. \tag{14.10}$$

Evaluate at $r = 10$ and $r = 15$, this gives us $\ell = 128$ and $k = 256$ when $r = 10$ going to $\ell = 58.89$ and $k = 75.85$ when r increases to 15.

Exercise 14B.12 *Verify these calculations.*

Answer: Setting short run market supply equal to market demand implies

$$417,586p^{2/3} = \frac{40,000,000}{p^2}, \tag{14.11}$$

which implies $p^{8/3} = 95.789$ or $p = 5.533409 \approx \$5.53$. Substituting this into the firm's new short run supply function $x^{s'}(p) = 334.069p^{2/3}$, we get that each firm produces $xs'(5.53) \approx 1,045$. The industry output can be calculated by substituting the new price into either the market demand or supply curves — both of which tell us that overall industry output will rise to 1,306,395.[1] Total revenue for each firm will then simply be the price times the output level 1,045 — or approximately \$5,782 if we use the un-rounded price or \$5,779 if we use $p = 5.53$ which is slightly rounded down.

[1] Because of rounding error, you will actually get slightly different answers depending on whether you plug the new price into the market demand or short run market supply functions — the output level 1,306,395 arises from using the price $p = 5.533409$ we calculated before rounding. Up to a rounding error, this is also the same as what we get if we multiply each firm's output of 1,045 by the total number of firms in the short run equilibrium (1,250).

Exercise 14B.13 *How much does the industry production change in the short run?*

Answer: Industry production falls from the original 1,600,000 calculated earlier to the 1,306,395 we calculated in the previous exercise — a short run drop of a little less than 300,000 output units.

Exercise 14B.14 *Verify these calculations and compare the results to our graphical analysis of an increase in the wage rate in Graph 14.10.*

Answer: First, to calculate the long run equilibrium price, we need to determine the lowest point of the long run average cost curve. The cost curve $C(w,r,x) = 2(wr)^{1/2}(x/20)^{5/4} + 1280$ (given at the beginning of part B of the text) becomes $C(30,10,x) = 0.819036x^{5/4} + 1280$ when the new wage (and original rental rate) are substituted for w and r. Thus, the average (long run) cost curve after the wage increase is

$$AC(x) = 0.819036x^{1/4} + \frac{1280}{x}. \tag{14.12}$$

This reaches its lowest point when

$$\frac{\partial AC(x)}{\partial x} = \frac{0.204759}{x^{3/4}} - \frac{1280}{x^2} = 0. \tag{14.13}$$

Solving for x, we get that $x = 1088.36 \approx 1088$ — and the lowest average cost level reached at that output level is $AC(1088.36) \approx \$5.88$. Thus, the price rises from the original \$5.00 to \$5.53 in the short run to \$5.88 in the long run. Each firm, which originally produced 1280 units, reduces its output to 1045 in the short run, but that output level increases to 1088 in the long run for those firms that remain in the industry. At the new long run equilibrium price, the market demands $D^M = 40,000,000/(5.88^2) \approx 1,156,925$ — down from the original 1,600,000 and from the short run equilibrium industry output of 1,306,395. This implies that the number of firms that remain in the industry falls from the original 1,250 to 1,156,925/1088≈ $1,063$ firms.

The graph in part A predicted that the industry would produce less in the short run and even less in the long run, and that the price will rise in the short run and even more in the long run. Both these predictions hold up in this example. The graph furthermore predicted that output by each firm will initially fall in the short run but will rise back to its original quantity in the long run for firms that stay in the industry. The latter does not hold in this example, but we had pointed out in part A that the long run output could in principle go up or down — and we simply graphed it as unchanged from the original quantity solely for convenience. Thus, the prediction in this example that firm output for those that remain in the industry will initially go down and then recover somewhat but not fully is not inconsistent with the discussion surrounding the graph in part A.

Exercise 14B.15 *How much does individual consumption by consumers who were originally in the market change in the short run?*

Answer: Individual demand was derived at the beginning of the chapter to be $x^d(p) = 625/p^2$. Substituting in $p = 5.91$, we then get that $x^d(5.91) = 625/(5.91^2) = 17.894 \approx 18$.

Exercise 14B.16 *Verify these calculations and compare the results with Graph 4.11 where we graphically illustrated the impact of an increase in market demand.*

Answer: Since the average cost curves for firms have not changed, the long run price falls to the previously calculated lowest point of the AC curve — which is \$5. Thus, firms will enter until price falls from the short run equilibrium of \$5.91 to the long run equilibrium of \$5. To meet market demand of 2,500,000 at that price, the new number of firms in the industry must be $2,500,000/1280 \approx 1,953$, up from the initial 1,250, with each firm producing at its original equilibrium quantity of 1,280 units of output. Thus, initially industry quantity rises in the short run because each firm produces more at a higher price, but in the long run each firm returns to its original quantity, more firms enter and price falls to its original level, with the industry increasing production beyond the short run increase. This is exactly what is demonstrated in Graph 14.11 of the text.

14C Solutions to End-of-Chapter Exercises

Exercise 14.1

In Table 14.1, the last column indicates the predicted change in the number of firms within an industry when economic conditions change.

A: *In two cases, the table makes a definitive prediction, whereas in two other cases it does not.*

(a) *Explain first why we can say definitively that the number of firms falls as a fixed cost (i.e. license fee) increases? Relate your answer to what we know about firm output and price in the long run.*

Answer: When a fixed cost increases, the long run MC curve does not change but the long run AC curve shifts up. Since the MC curve always crosses the lowest point of the AC curve, we know that this implies that the lowest point of the long run AC curve shifts to the right — i.e. to a higher level of output. This implies that the output level of firms that remain in the industry will increase in the new equilibrium — as will the price (since the AC curve has shifted up). But an increase in price means that, for any downward sloping demand curve, consumers will demand less of the good. The industry therefore produces less at a higher price — with each firm in the industry producing more. The only way this is possible is if some firms have exited — i.e. the number of firms in the industry has decreased.

(b) *Repeat (a) for the case of an increase in demand.*

Answer: When demand increases, none of the cost curves for firms change — with the long run AC curve in particular remaining unchanged. Thus, each firm in the new equilibrium will be producing at the same lowest point of its AC curve — and at the same price. The only thing that has changed is that demand has shifted — which implies that, at the same price, more output will be produced in the industry. With each firm in the industry producing the same output quantity, the only way more can be produced in the industry is for more firms to have entered — i.e. the number of firms in the industry has increased.

(c) *Now consider an increase in the wage rate and suppose first that this causes the long run AC curve to shift up without changing the output level at which the curve reaches its lowest point. In this case, can you predict whether the number of firms increases or decreases?*

Answer: In this case, the output level produced by each firm in the industry will remain the same but will be sold at a higher price. When price increases, however, less will be demanded (assuming a downward sloping market demand curve) — which implies the industry produces less. With each firm in the industry producing the same as before, the only way for the industry to produce less is for some firms to have exited. Thus, the number of firms in the industry falls in this case.

(d) *Repeat (c) but assume that the lowest point of the AC curve shifts up and to the right.*

Answer: If the lowest point of the *AC* curve shifts up and to the right, it means that firms that remain in the industry will produce *more* at a higher price — but the higher price implies that less will be demanded and thus the industry produces *less*. The only way each firm can produce more while the industry produces less is if some firms exited — and the number of firms in the industry therefore declines.

(e) *Repeat (c) again but this time assume that the lowest point of the AC curve shifts up and to the left.*

Answer: In this case, the lowest point of the *AC* curve occurs at a lower level of output and higher price — which means that firms in the industry will produce less and sell at that higher price. A higher price in turn means that consumers will demand less. Thus, each firm produces *less* as does the industry. Whether this implies more or fewer firms now depends on how much less each firm produces relative to how much the quantity demanded falls with the increase in price. Suppose each firm produces x% less and the industry as a whole produces y% less. Then if $x = y$, the number oof firms stays exactly the same; if $x < y$, the number of firms falls and if $x > y$, the number of firms in the industry has to increase.

(f) *Is the analysis regarding the new equilibrium number of firms any different for a change in r?*

Answer: No, the analysis is no different for a change in r. Even if capital is fixed in the short run, it is variable in the long run — and treated just like the input labor.

(g) *Which way would the lowest point of the AC curve have to shift in order for us not to be sure whether the number of firms increases or decreases when w* falls*?*

Answer: When w falls, we know the long run *AC* curve will shift down — which implies that the long run equilibrium price will fall. At a lower price, the quantity demanded will increase — which implies that industry output will *increase*. Were each firm to continue to produce the same amount as before — or were each firm to produce less, then the only way for the industry to produce more would be for the number of firms to increase. Thus, in order for us not to be sure of whether the number of firms increases, it would have to be that each firm produces more (just as the industry produces more) — and this only happens if the lowest point of the *AC* curve shifts to the right as w falls.

B: *Consider the case of a firm that operates with a Cobb-Douglas production function $f(\ell,k) = A\ell^{\alpha}k^{\beta}$ where $\alpha,\beta > 0$ and $\alpha+\beta < 1$.*

(a) *The cost function for such a production process — assuming no fixed costs — was given in equation (13.45) of exercise 13.5. Assuming an additional recurring fixed cost F, what is the average cost function for this firm?*

Answer: Including the fixed cost F, the total cost function becomes

$$C(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + F \tag{14.14}$$

which gives us an *AC* function

$$AC(w,r,x) = \frac{C(w,r,x)}{x} = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{F}{x}. \tag{14.15}$$

(b) *Derive the equation for the output level x^* at which the long run AC curve reaches its lowest point.*

Answer: The AC curve reaches its lowest point where its derivative with respect to x is zero — i.e. where

$$\frac{\partial AC(w,r,x)}{\partial x} = \left[(1-\alpha-\beta)\left(\frac{w^\alpha r^\beta}{A\alpha^\alpha \beta^\beta}\right)^{1/(\alpha+\beta)} x^{(1-2(\alpha+\beta))/(\alpha+\beta)}\right] - \frac{F}{x^2} = 0. \tag{14.16}$$

Solving this for x, we then get the output level at the lowest point of the long run AC curve:

$$x^* = \left(\frac{A\alpha^\alpha \beta^\beta}{w^\alpha r^\beta}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(\alpha+\beta)}. \tag{14.17}$$

(c) *How does x^* change with F, w and r?*

Answer: Given the expression for x^* above, it is easy to see that x^* increases with F and decreases with w and r. Put differently, the lowest point of the AC curve occurs at higher output levels as the fixed cost increases and at lower output levels when input prices increase.

(d) True or False*: For industries in which firms face Cobb-Douglas production processes with recurring fixed costs, we can predict that the number of firms in the industry increases with F but we cannot predict whether the number of firms will increase or decrease with w or r.*

Answer: This is true. As F increases, output price rises as does output by each firm. The higher output price, however, means that the quantity demanded — and thus the quantity supplied by the industry — decreases. The only way the industry output can decrease when firm output increases is if some firms have left the industry. When input prices increase, the equilibrium output price similarly rises (as the AC shifts up) — causing the industry to produce less. But, in the case analyzed here, each firm also produces less — so we cannot immediately tell whether the number of firms will increase or decrease.

Exercise 14.4: *Brand Names and Franchise Fees*

Business Application: *Brand Names and Franchise Fees*: *Suppose you are currently operating a hamburger restaurant that is part of a competitive industry in your city.*

A: *Your restaurant is identical to others in its homothetic production technology which employs labor ℓ and capital k and has decreasing returns to scale.*

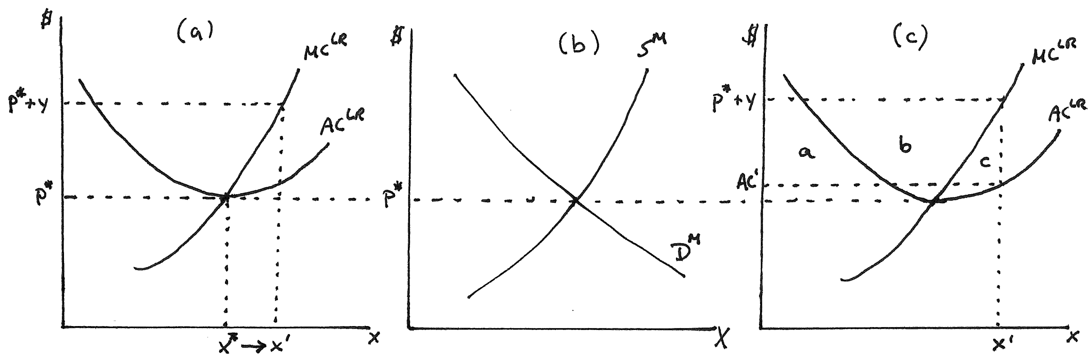
(a) *In addition to paying for labor and capital each week, each restaurant also has to pay recurring weekly fees F in order to operate. Illustrate the average weekly long run cost curve for your restaurant.*

Answer: This is illustrated in panel (a) of Graph 14.6 where the long run average cost curve for your restaurant is U-shaped because of the combination of a recurring fixed cost and decreasing returns to scale in production.

(b) *On a separate graph, illustrate the weekly demand curve for hamburgers in your city as well as the short run industry supply curve assuming that the industry is in long run equilibrium. How many hamburgers do you sell each week?*

Answer: This is illustrated in panel (b) where the market supply curve S^M is simply all short run restaurant supply curves added together. This has to intersect demand at p^* which lies at the lowest point of the AC curve in panel (a). It is only at that price that long run profits for restaurants are zero and thus no incentives for entering or exiting the industry exist. At this price, you will sell hamburgers so long as p^* is greater than long run average cost — and we know that MC crosses the AC curve at its lowest point. Thus, you will produce x^* as indicated in panel (a) of Graph 14.6.

(c) *As you are happily producing burgers in this long run equilibrium, a representative from the national MacWendy's chain comes to your restaurant and asks you to convert your restaurant to a MacWendy's. It turns out, this would require no effort on your part — you would*



Graph 14.6: MacWendy's Franchise Deal

simply have to allow the MacWendy's company to install a MacWendy's sign, change some of the furniture and provide your employees with new uniforms — all of which the MacWendy's parent company is happy to pay for. MacWendy's would, however, charge you a weekly franchise fee of G for the privilege of being the only MacWendy's restaurant in town. When you wonder why you would agree to this, the MacWendy's representative pulls out his marketing research that convincingly documents that consumers are willing to pay $y more per hamburger when it carries the MacWendy's brand name. If you accept this deal, will the market price for hamburgers in your city change?

Answer: The market price for hamburgers in your city would remain unchanged at p^* since the industry is competitive and thus a single firm's actions cannot change prices. However, the price you can charge for a MacWendy's hamburger will be $p^* + y$.

(d) *On your average cost curve graph, illustrate how many hamburgers you would produce if you accepted the MacWendy's deal.*

Answer: You would produce where $p^* + y$ intersects your marginal cost curve. In the long run, we would use the long run marginal cost curve which is illustrated in panel (a) of Graph 14.6. Output at your restaurant would then increase from x^* to x'. (If capital is fixed in the short run, your initial increase in output would be less since the short run MC curve is steeper — and you would fully adjust to x' only once you can adjust your level of capital.)

(e) *Next, for a given y, illustrate the largest that G could be in order for you to accept the deal offered by MacWendy's.*

Answer: This is done in panel (c) of Graph 14.6 where the long run average and marginal cost curves are drawn once again. We know that you will be able to sell your MacWendy's hamburgers at the price $p^* + y$, and we know you will sell x'. That makes your revenue equal to the box $(p^* + y)x'$. We also know you will incur average costs AC' — or total long run costs AC' times x', the smaller rectangular box. The difference between these two boxes $(a+b+c)$ is the long run profit that you can make (per time period) from being a MacWendy's *not counting the franchise fee G*. Thus, $a + b + c$ is the most you would be willing to pay per time period for the franchise fee.

(f) *If you accept the deal, will you end up hiring more or fewer workers? Will you hire more or less capital?*

Answer: Since neither the wage nor the rental rate has changed, and since the production technology is homothetic, we know your labor to capital ratio will not change as you produce more output (because all cost minimizing input bundles lie on the same ray from the origin in the isoquant graph). We know from what we did above that you will produce more — thus you will hire more labor and more capital.

(g) *Does your decision on how many workers and capital to hire under the MacWendy's deal depend on the size of the franchise fee G?*

Answer: No — once you accept the Wendy's deal, it is a fixed cost that has no impact on the MC curve. It therefore has no impact on how much you will produce — and thus no impact on how many workers and capital you hire.

(h) *Suppose that you accepted the MacWendy's deal and, because of the increased sales of hamburgers at your restaurant, one hamburger restaurant in the city closes down. Assuming that the total number of hamburgers consumed remains the same, can you speculate whether total employment (of labor) in the hamburger industry went up or down in the city? (*Hint*: Think about the fact that all restaurants operate under the same decreasing returns to scale technology.)*

Answer: If my increased production drove one restaurant out of business and the overall number of hamburgers sold in the city remains unchanged, it must mean that production by the other restaurant that was driven out of business went down by x^* while your production went from x^* to $2x^*$. With decreasing returns to scale, however, the other restaurant needed to use less labor and capital to produce x^* than you need to use to increase your output from x^* to $2x^*$. Thus, overall employment in the restaurant sector of the city increases.

B: *Suppose all restaurants in the industry use the same technology that has a long run cost function $C(w,r,x) = 0.028486(w^{0.5}r^{0.5}x^{1.25})$ which, as a function of wage w and rental rate r, gives the weekly cost of producing x hamburgers.*[2]

(a) *Suppose that each hamburger restaurant has to pay a recurring weekly fee of \$4,320 to operate in the city in which you are located and that $w = 15$ and $r = 20$. If the restaurant industry is in long run equilibrium in your city, how many hamburgers does each restaurant sell each week?*

Answer: If the industry is in long run equilibrium, each restaurant makes zero long run profit and thus operates at the lowest point of its AC curve. Given $w = 15$ and $r = 20$, the average cost function is

$$AC(x) = \frac{0.028486(15^{0.5}20^{0.5}x^{1.25})}{x} + \frac{4320}{x} = 0.4934x^{0.25} + \frac{4320}{x}. \tag{14.18}$$

The lowest point of this AC curve occurs where the derivative with respect to x is zero — i.e. where

$$\frac{dAC(x)}{dx} = \frac{0.12335}{x^{0.75}} - \frac{4320}{x^2} = 0. \tag{14.19}$$

Solving this for x, we get $x = 4320$. Thus, each restaurant sells 4,320 hamburgers per week.

(b) *At what price do hamburgers sell in your city?*

Answer: Plugging 4,320 into the AC function in equation (14.18), this gives us the lowest point of the AC curve on the vertical axis — which is also equal to the long run equilibrium price. That price is $p = 5$.

(c) *Suppose that the weekly demand for hamburgers in your city is $x(p) = 100{,}040 - 1000p$. How many hamburger restaurants are there in the city?*

Answer: At a price of \$5 per hamburger, the total demand will be $x = 100{,}040 - 1000(5) = 95{,}040$ hamburgers per week. With each hamburger restaurant producing 4,320 per week, this implies that the number of such restaurants in the city is $95040/4320 = 22$.

[2] For those who find unending amusement in proving such things, you can check that this cost function arises from the Cobb-Douglas production function $f(\ell,k) = 30\ell^{0.4}k^{0.4}$.

(d) *Now consider the MacWendy's offer described in A(c) of this exercise. In particular, suppose that the franchise fee required by MacWendy's is $G = 5,000$ and that consumers are willing to pay 94 cents more per hamburger when it carries the MacWendy's brand name. How many hamburgers would you end up producing if you accept MacWendy's deal?*

Answer: Since you would be able to sell your MacWendy's hamburgers for \$5.94 instead of \$5, we need to determine how many you would produce from your long run marginal cost curve. Given the cost function $C(w,r,x) = 0.028486(w^{0.5}r^{0.5}x^{1.25})$ that becomes $C(x) = 0.493392x^{1.25}$ when evaluated at the input prices $w = 15$ and $r = 20$, this is

$$MC(x) = \frac{\partial C(x)}{\partial x} = 0.61674x^{0.25}. \tag{14.20}$$

(Note that the fixed cost is irrelevant for the marginal cost curve which is why we did not need to include it in the $C(x)$ function we differentiated.) You will produce until price is equal to marginal cost — i.e. until $p = 5.94 = 0.61674x^{0.25}$. Solving this for x, we get that you will produce $x = 8,605$ hamburgers per week.

(e) *Will you accept the MacWendy's deal?*

Answer: Yes, you will — because your long run profit is now positive. We just determined that you will sell 8,605 hamburgers per week at a price of \$5.94 — which gives you total revenue of about \$51,114. Your weekly cost is given by the cost function that includes the fixed cost and franchise fee:

$$C = 0.493392\left(8605^{1.25}\right) + 4320 + 5000 \approx 50,211. \tag{14.21}$$

Subtracting costs from revenues, we get a long run profit of \$903 per week.

(f) *Assuming that the total number of hamburgers sold in your city will remain roughly the same, would the number of hamburger restaurants in the city change as a result of you accepting the deal?*

Answer: Since you are selling roughly twice as many hamburgers (8,605 versus 4,320) as a MacWendy's hamburger restaurant, you will in effect drive one restaurant out of the market. The total number of hamburger restaurants therefore falls to 21 — 20 of the general kind and 1 MacWendy's.

(g) *What is the most that the MacWendy's representative could have charged you for you to have been willing to accept the deal?*

Answer: Since you are making \$903 in weekly profit when you are paying a \$5,000 weekly franchise fee, the most that the MacWendy's representative could have charged is \$5,903 per week.

(h) *Suppose the average employee works for 36 hours per week. Can you use Shephard's Lemma to determine how many employees you hire if you accept the deal? Does this depend on how high a franchise fee you are paying?*

Answer: Applying Shephard's Lemma to the function $C(w,r,x) = 0.028486(w^{0.5}r^{0.5}x^{1.25})$, we get the conditional labor demand function

$$\ell(w,r,x) = \frac{\partial C(w,r,x)}{\partial w} = 0.014243\left(\frac{r^{0.5}x^{1.25}}{w^{0.5}}\right). \tag{14.22}$$

After becoming a MacWendy's, you are producing 8,605 hamburgers per week. Plugging in $x = 8605$ and the input prices $w = 15$ and $r = 20$, we therefore get that you will hire

$$\ell = 0.014243\left(\frac{20^{0.5}8605^{1.25}}{15^{0.5}}\right) \approx 1,363 \tag{14.23}$$

hours of labor. With the average employee working 36 hours per week, this implies 37.86 workers. Note that the franchise fee is irrelevant to this because it is a fixed cost — as long as you accept it, you will hire about 38 workers.

(i) *How does this compare to the number of employees hired by the competing non-MacWendy's hamburger restaurants? In light of your answer to (f), will overall employment in the hamburger industry increase or decrease in your city as a result of you becoming a MacWendy's restaurant?*

Answer: Your competitors face the same input prices $w = 15$ and $r = 20$ but produce only 4,320 hamburgers per week. Plugging these values into equation (14.22), we get

$$\ell = 0.014243\left(\frac{20^{0.5}4320^{1.25}}{15^{0.5}}\right) = 576 \qquad (14.24)$$

hours of labor per week. At an average work week of 36 hours, this implies 16 workers per week. Given that the number of restaurants will decrease by 1 as a result of you becoming a MacWendy's, the city will lose 16 hamburger restaurant jobs — but it will gain about 22 jobs in your MacWendy's (since you employed 16 workers before becoming a MacWendy's and about 38 afterwards). Thus, there is a net increase of 6 hamburger restaurant jobs in the city as a result of you becoming a MacWendy's restaurant.

Exercise 14.10: *School Vouchers and Private School Markets*

Policy Application: *School Vouchers and the Private School Market: In the U.S., private schools charge tuition and compete against public schools that do not. One policy proposal that is often discussed involves increasing demand for private schools through* school vouchers. *A school voucher is simply a piece of paper with a dollar amount V that is given to parents who can pay for some portion of private school tuition with the voucher if they send their child to a private school. (Private schools can then redeem the vouchers for a payment of V from the government.) Assume throughout that private schools strive to maximize profit.*

A: *Suppose private schools have U-shaped average (long run) cost curves, and the private school market in a metropolitan area is currently in long run equilibrium (in the absence of private school vouchers).*

(a) *Begin by drawing a school's average long run cost curve (with the number of private school seats on the horizontal axis). Then, in a separate graph next to this, illustrate the city-wide demand curve for seats in private schools as a function of the tuition price p. Finally, include the short run aggregate supply curve that intersects with demand at a price that causes the private school market to be in long run equilibrium.*

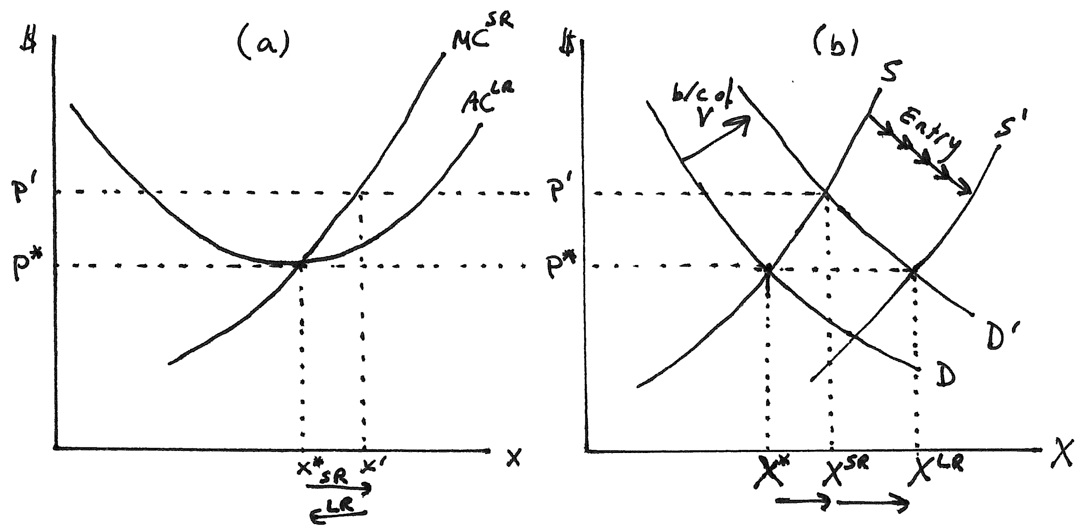
Answer: This is done in Graph 14.7. In order for the private school market to be in long run equilibrium, each private school makes zero profit and thus operates on the lowest point of its long run AC curve. This implies that our initial supply S and demand D must intersect at price p^* and each private school educates x^* children.

(b) *Illustrate what happens to the demand curve as a result of the government making available vouchers in the amount of V to all families who live in the city. What happens to the number of seats made available in each existing private school, and what happens to the tuition level p in the short run?*

Answer: In panel (b) of Graph 14.7, we illustrate a shift out in the demand curve as parents are now more willing to pay for private schools. This shift in demand causes a short run increase in price to p' — with each school providing x' seats. Thus each school will open more seats in the short run at a higher tuition level, and more children in the city will attend private schools.

(c) *Next, consider the long run when additional private schools can enter the market. How does the tuition level p, the number of seats in each school and the overall number of children attending private schools change?*

Answer: In the short run, existing private schools will make profit as they operate at a price p' that lies above the lowest point on their AC curves. This will induce entry into the private school market — with new private schools entering. And this will in turn cause the short run



Graph 14.7: Private School Markets and School Vouchers

supply curve in panel (b) of Graph 14.7 to shift out with each new entrant. The entry of new schools will continue so long as there are still positive profits in the private school market — i.e. so long as price remains above the original price p^*. (This assumes that entering schools have the same cost structure as existing schools — if they are less efficient, the process of entry would end before p^* is reached.) In the long run, with price falling back to p^*, we would then expect each private school to go back to supplying seats for x^* children, but the private school market will increase its supply of seats from the short run quantity of X^{SR} to X^{LR} (in panel (b) of the graph).

(d) *Opponents of private school vouchers sometimes express concern that the implementation of vouchers will simply cause private schools to increase their tuition level and thus cause no real change in who attends private school. Evaluate this concern from both a short and long run perspective.*

Answer: This concern is more valid in the short run than in the long run, but even in the short run it is not entirely correct. As we showed, it is certainly the case that tuition would increase in the short run — but we also showed that the number of children attending private schools would increase. So there would, even in the short run, be a real change in terms of who goes to school where — with the size of the change depending on just how steep short run MC curves are. In the long run, the concern appears largely invalid because — if schools can indeed increase tuition in response to voucher-induced demand, new schools will enter the market and eventually drive tuition price back down to its original level.

(e) *Proponents of private school vouchers often argue that the increased availability of private schools will cause public schools to offer higher quality education. If this is true, how would your answers to (b) and (c) change as a result?*

Answer: If proponents are correct, then parental demand for private schools should fall as private school competition increases and improves public schools. Thus, in the short run tuition levels would not rise as much. In the long run, tuition levels would fall back to p^*, but the overall increase of the market would be smaller (i.e. fewer private schools would enter.)

(f) *If private school vouchers are made available to anyone who lives within the city boundaries (but not to those who live in suburbs), some families who previously chose to live in suburbs to send their children to suburban public schools might choose instead to live in the city and send their children to private schools. How would this affect your answers to (b) and (c)?*

Answer: This would exacerbate the increase in demand from the voucher as this demand would now come not only from parents in cities but also from parents that move to the city because of vouchers. Although it seems unlikely that such relocations of families will occur in the short run, to the extent that it does it would cause the short run increase in private school tuitions to be larger than otherwise predicted. In the long run, tuition prices would still have to fall to p^* due to entry of new schools — but the overall size of the private school market would now be larger than otherwise predicted (as more firms enter in response to the larger shift in demand.)

B: *In the following, all dollar values are expressed in thousands of dollars. Suppose that the total city-wide demand function for private school seats x is given by $x(p) = 24710 - 2500p$ and each private school's average long run cost function is given by $AC(x) = 0.655x^{1/3} + (900/x)$.*

(a) *Verify that $AC(x)$ arises from a Cobb-Douglas production function $x = f(\ell, k) = 35\ell^{0.5}k^{0.25}$ when $w = 50$ and $r = 25$ and when private schools face a fixed cost of 900. One unit of x is interpreted as one seat (or one child) in the school, and ℓ is interpreted here as a teacher. (Since dollar values are expressed in thousands, w represents a teacher's salary of $50,000 and the fixed cost represents a recurring annual cost of $900,000.)*

Answer: In Chapter 13 end of chapter exercises, we derived the AC function for a Cobb-Douglas production function of the form $f(\ell, k) = A\ell^{\alpha}k^{\beta}$ when the firm faces a fixed cost of FC as

$$AC(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{FC}{x}. \tag{14.25}$$

Substituting in $\alpha = 0.5$, $\beta = 0.25$, $A = 35$, $w = 50$, $r = 25$ and $FC = 900$, we get the average cost function specified in the problem.

(b) *In order for the private school market to be in long run equilibrium, how many children are served in each private school? What is the tuition (per seat in the school) charged in each private school?*

Answer: The tuition has to settle at the lowest point of the AC curve — i.e. the point at which its slope is zero. We can thus take the first derivative of $AC(x)$ and set it to zero, which gives

$$\frac{dAC(x)}{dx} = \frac{1}{3}0.655x^{-2/3} - 900x^{-2} = 0. \tag{14.26}$$

Solving this for x, we get $x \approx 515$ — the number of children served in each private school. The average cost with this many children is then

$$AC(515) = 0.655(515^{1/3}) + \frac{900}{515} \approx 7. \tag{14.27}$$

The price for tuition in the private school market is therefore 7 — or $7,000 per student (since dollar amounts are in thousands.)

(c) *Given that you know the underlying production function, can you determine the class size in each private school? (*Hint*: You already determined the total number of children in part (a) and now need to determine the number of teachers in each private school.)*[3]

Answer: In chapter 13 end-of-chapter exercises, we determined that the long run labor demand function for a Cobb-Douglas production process $f(\ell, k) = A\ell^{\alpha}k^{\beta}$ is

$$\ell(w,r,p) = \left(\frac{pA\alpha^{(1-\beta)}\beta^{\beta}}{w^{(1-\beta)}r^{\beta}}\right)^{1/(1-\alpha-\beta)}. \tag{14.28}$$

[3] It may be helpful to check equation (13.50) in exercise 13.8.

Substituting in $\alpha = 0.5$, $\beta = 0.25$, $A = 35$, $w = 50$, $r = 25$ and $p = 7$, we get $\ell \approx 36$. Thus, each private school has 36 teachers and admits about 515 students — giving us a class size of $515/36 \approx 14.3$ students per teacher.

(d) *How many private schools are operating?*

Answer: At a tuition price of $p = 7$, we can determine total demand from the demand function:

$$x = 24710 - 2500(7) = 7{,}210. \tag{14.29}$$

With each school serving about 515 students, this implies that there must be $7210/515 = 14$ private schools in the market.

(e) *Now suppose that the government makes private school vouchers in the amount of 5.35 (i.e. \$5,350) per child available to parents. How will this change the demand function for seats in private schools? (*Hint*: Be careful to add the voucher in the correct way — i.e. to make the demand* curve *shift up.)*

Answer: This will shift the demand curve up by 5.35. The demand curve, however, has price on the vertical and x on the horizontal — so we need to write it as a function of x rather than as a function of p in order to add 5.35 to it. Taking the demand function $x(p) = 24710 - 2500p$ and solving for p, we get

$$p(x) = \frac{24710}{2500} - \left(\frac{1}{2500}\right)x = 9.884 - 0.0004x. \tag{14.30}$$

We can now add 5.35 to this given that the voucher will shift the demand curve up by this amount — which gives us $p(x) = 15.234 - 0.0004x$. Solving back for x, we can then get the new demand function

$$x(p) = 38{,}085 - 2500p. \tag{14.31}$$

(f) *Given this change in demand, what will happen to tuition and the number of children served in existing private schools in the short run assuming the number of schools is fixed and no new schools can enter in the short run? (*Hint*: You will need to know the current level of capital, derive the short run supply function for private schools, then aggregate them across the existing private schools.)*

Answer: In chapter 13 end-of-chapter exercises, we determined that the long run labor demand function for a Cobb-Douglas production process $f(\ell, k) = A\ell^{\alpha}k^{\beta}$ is

$$k(w,r,p) = \left(\frac{pA\alpha^{\alpha}\beta^{(1-\alpha)}}{w^{\alpha}r^{(1-\alpha)}}\right)^{1/(1-\alpha-\beta)}. \tag{14.32}$$

Substituting in $\alpha = 0.5$, $\beta = 0.25$, $A = 35$, $w = 50$, $r = 25$ and $p = 7$, we get $k \approx 36$. When voucher are introduced, each private school therefore has $\overline{k} = 36$ units of capital that are fixed in the short run. The short run production function is then

$$x = f_{\overline{k}}(\ell) = \left[35\left(36^{0.25}\right)\right]\ell^{0.5} \approx 85.73\ell^{0.5}. \tag{14.33}$$

The short run profit maximization problem is then

$$\max_{\ell}\ p\left(85.73\ell^{0.5}\right) - 50\ell. \tag{14.34}$$

Solving this, we get the short run labor demand function, and substituting it back into equation (14.33), we get the short run supply function:

$$\ell_{\overline{k}}(p) = 0.735p^2 \ \text{ and } \ x_{\overline{k}}(p) = 73.5p. \tag{14.35}$$

Since there are 14 private schools in the pre-voucher long run equilibrium, the aggregate short run supply function is then

$$X^{SR}(p) = 14(73.5p) = 1{,}029p. \tag{14.36}$$

We can now find the short run equilibrium price by setting the demand function (that includes the voucher) from equation (14.31) equal to this short run aggregate supply function; i.e.

$$38{,}085 - 2500p = 1{,}029p \tag{14.37}$$

and solve for p. This gives us the short run equilibrium tuition price of $p = 10.792$, which is (given we are measuring prices in thousands) \$10,792, up from the initial \$7,000 pre-voucher price.

The number of children served can now be determined from the demand function that includes the voucher (equation (14.31)) by substituting in $p = 10.792$ — which gives us

$$x = 38085 - 2500(10.792) \approx 11{,}105. \tag{14.38}$$

This is the total number of children in private schools in the short run equilibrium. Since the number of schools is unchanged at 14 in the short run, there will be $11{,}105/14 \approx 793$ students per school, up from the initial 515. (You can also verify this by plugging the short run equilibrium price $p = 10.792$ into the school's short run supply function in equation (14.35) to get $x_{\overline{k}} = 73.5(10.792) \approx 793$.)

(g) *What happens to private school class size in the short run?*

Answer: We already know from the previous part that the number of children in each private school will be 793 in the short run. To determine class size, we just have to determine the number of teachers working in each school in the short run. For this, we can use the short run labor demand function from equation (14.35), substitute the short run equilibrium price $p = 10.792$ and get

$$\ell_{\overline{k}} = 0.735p^2 = 0.735(10.792)^2 \approx 85.6. \tag{14.39}$$

With an average of 85.6 teachers and 793 students per school, class size in the short run falls to $793/85.6 \approx 9.3$ (from the original 14.3).

(h) *How do your answers change in the long run when new schools can enter?*

Answer: In the long run, new schools will enter the private school market until each school again operates at the lowest point of its long run average cost curve (and thus makes zero profit). This means that the tuition price will be driven back down to $p = 7$ in the new long run equilibrium, with each private school again serving about 515 students. (We calculated this in part (a) where we simply used the long run AC curve (which has not changed) to determine these values.) The number of teachers in each school will again go to 36, giving us a long run equilibrium class size of 14.3. But because of the voucher, more children will be attending private schools. We can determine exactly how many more by using the demand function that includes the voucher (equation (14.31)) and substituting in the long run equilibrium price $p = 7$ to get

$$x = 38085 - 2500(7) = 20{,}585. \tag{14.40}$$

With 515 students per school, this implies the market will expand to 40 schools (from the initial 14) as a result of new schools forming to meet increased demand.

Conclusion: Potentially Helpful Reminders

1. Students often find it irritating that we have defined the "short run" differently for firms than for industries — with firms operating in the "short run" as long as their capital level is fixed, and industries operating in the "short

run" so long as entry and exit of firms is not possible. You might be less irritated if you recognize that the two definitions actually boil down to the same definition: A firm needs to be able to invest in capital in order to enter (or to get rid of its capital in order to exit) the industry. Thus, a firm can enter or exit only if it can vary capital — which links our two ways of thinking about the "short run".

2. Of all the firm cost and expenditure curves we derived in the previous three chapters, only two are crucial in this chapter: (1) The long run *AC* curve (of the marginal firm) — or, to be even more specific, the lowest point of that long run *AC* curve. This determines long run price. (2) The short run *MC* curve for the fixed level of capital each firm has — which gives rise to the short run supply curve.

3. We often start out our analysis by assuming that an industry is initially in long run equilibrium. That implies that we start with a picture in which the marginal firm's short run supply curve crosses its long run *AC* curve at the lowest point of the *AC* curve. It extends below the *AC* curve because the short run shut-down price lies below the long run exit price. *Avoid drawing unnecessary curves that don't matter for your analysis.*

4. When you analyze a change that occurs within an industry that is initially in long run equilibrium, you therefore start with the initial long run picture of the marginal firm (with its long run *AC* curve and the short run supply curve) — and then ask whether any of the changes altered either of these curves. Then you can trace out what happens in the short run and the long run.

5. You can conclude what happens to the overall number of firms in the industry if you know what happens to each firm's output when we go from the initial to the new long run equilibrium and you know whether the overall quantity demanded at the new long run equilibrium price is higher or lower than it was originally. (The answer to the question of whether the number of firms increases or decreased will often be ambiguous if the change affecting the industry is a change in input prices). In the short run, the number of firms is fixed.