CHAPTER

15

The "Invisible Hand" and the First Welfare Theorem

Chapter 14 introduced the *positive* idea of equilibrium in the context of a competitive environment — and Chapter 15 now moves onto the more *normative* assessment of a competitive equilibrium within the context of the first welfare theorem. Put differently, Chapter 14 focuses on *predicting* changes in economic environments in competitive settings while Chapter 15 now focuses on *welfare* as defined by consumer surplus and profit (or producer surplus). In Chapter 14 the consumer side of the market did not play a prominent role — we simply said that the market demand curve arises from the sum of individual demands. This is all we need for prediction. In the Chapter 15, on the other hand, we return to some themes from consumer theory — particularly the insight that welfare is measured on marginal willingness to pay (or compensated demand) curves and that these are the same as regular (or uncompensated) demand curves (that we use for prediction) only in the case of quasilinear tastes.

Chapter Highlights

The main points of the chapter are:

1. It is generally not possible to interpret curves that emerge from aggregating individual consumer demand (or labor supply) curves as if they emerged from an individual's optimization problem. Interpreting aggregate economic relationships that emerge from utility maximization in such a way is possible only if redistributing resources within the aggregated group leads to individually offsetting changes in behavior — i.e. **offsetting income effects**.

2. **It is possible to treat aggregate (or market) demand curves as if they emerged from an individual optimization problem** if there are no income effects — i.e. **if the good of interest is quasilinear**. In that special case, (uncompensated) demand curves are also equal to marginal willingness to pay (or com-

pensated demand) curves, enabling us to **measure consumer surplus on the market demand curve**.

3. Since economic relationships emerging from profit maximization by firms do not involve income effects, there are **no analogous issues with interpreting aggregate or market supply curves** (or labor demand curves) as if they emerged from a single optimization problem. As a result, we can measure **producer surplus (or profit) on the market supply curve** without making any particular assumptions.

4. Under a certain set of conditions, market equilibrium leads to output levels that mirror what would be chosen by omniscient social planners that aim to maximize overall social surplus. This is known as the **first welfare theorem** of economics which **specifies the conditions under which markets allocate resources efficiently**.

5. The advantage of market allocations of resources is that they rely on the **self-interested behavior by individuals who know only their own circumstances and observe the market price** signal that coordinates actions of producers and consumers. The disadvantages of market allocation of resources arise first in real world **violations of the assumptions underlying the first welfare theorem** that lead to violations of efficiency and second on normative judgments about **equity versus efficiency** that may lead us to conclude that some market outcomes, while being efficient, are in some sense "unfair" or "unjust".

Using the LiveGraphs

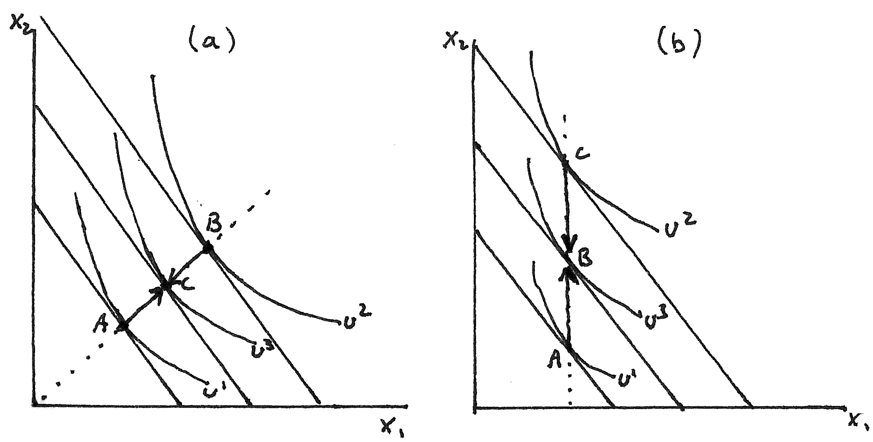
For an overview of what is contained on the LiveGraphs site for each of the chapters (from Chapter 2 through 29) and how you might utilize this resource, see pages 2-3 of Chapter 1 of this *Study Guide*. To access the LiveGraphs for Chapter 15, click the *Chapter 15* tab on the left side of the LiveGraphs web site.

We do not at this point have any additional *Exploring Relationships* exercises for this chapter but refer you back to some of those developed for consumer theory as it relates to the difference between uncompensated demand and marginal willingness to pay.

15A Solutions to Within-Chapter-Exercises for Part A

Exercise 15A.1 *Suppose that my tastes and my wife's tastes are exactly identical. If our tastes are also homothetic, does our household behave like a single representative agent? What if our* *tastes are quasilinear and neither individual is at a corner solution?*

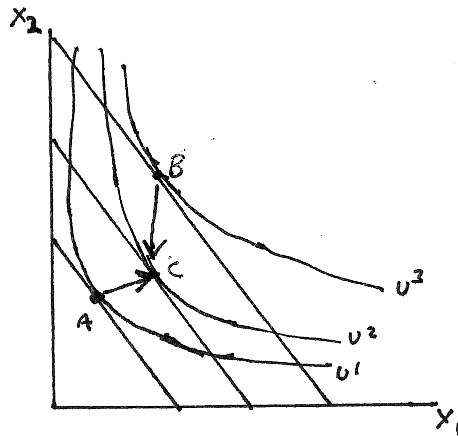
Answer: The answer is that, in both cases, our household will behave like an individual agent. This is illustrated in Graph 15.1. In panel (a), we assume that our tastes are homothetic and identical. This implies that both my wife and I will optimize along a ray from the origin, with the precise ray depending on the output prices (and thus the slope of the budget constraints). Suppose that I initially have the lowest of these budget constraints and my wife initially has the highest. Then I will optimize at A and she will optimize at B. If you then redistribute income so we both face the same budget constraint, we will both face the middle one — and we will both optimize at C. Thus, my wife's optimal bundle will move inward along the ray and mine will move outward along the ray — exactly offsetting each other. Our overall bundle will thus remain the same as you redistribute income. The same is true in panel (b) where our tastes are quasilinear and neither of us is at a corner solution.



Graph 15.1: Individual Agents when Tastes are Identical

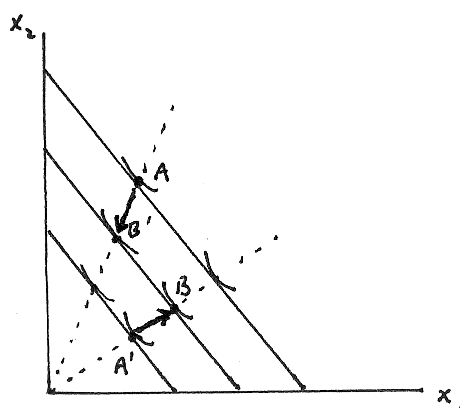
Exercise 15A.2 *Can you illustrate a case where our tastes are identical but we do not behave as a representative agent?*

Answer: One such case is illustrated in Graph 15.2 (next page). Let's assume that the three indifference curves are drawn from the same map of indifference curves — i.e. the same tastes. Initially I have the low income and my wife has the high income — which means I choose A and she chooses B. Then you redistribute income so we both face the middle income — and we both choose C. The change in my wife's consumption bundle is then clearly not offset by the change in mine — because the arrows are not parallel to one another.



Graph 15.2: Individual Agents when Tastes are Identical: Part 2

Exercise 15A.3 *Suppose both my wife and I have homothetic tastes but they are not identical. Does this still imply that we behave like a single representative agent?*



Graph 15.3: Homothetic Tastes

Answer: No, we will not behave as a representative agent (unless the tastes are over perfect substitutes). This is illustrated in Graph 15.3. Suppose my wife has tastes that cause her to optimize on the steeper ray from the origin and I have tastes that cause me to optimize on the shallower one. If initially I have the low income and she has the high income, she will choose A and I will choose A'. After you redistribute income and we both face the middle budget constraint, she will choose B' and I will choose B. Because we are moving along rays that have different slopes, the changes in our consumption bundles will not offset one another.

Exercise 15A.4 True or False*: As long as everyone has quasilinear tastes, the group will behave like a representative agent even if all the individuals do not share the same tastes (assuming no one is at a corner solution). The same is also true if everyone has homothetic tastes.*

Answer: The first part of the statement is true but the second part is false. For quasilinear tastes, the graph in the text in fact has me and my wife having *different* tastes that are quasilinear. So we have demonstrated in the text that the first part is true — as long as tastes are quasilinear, the group will act as an individual agent. In exercise 15A.3, however, we have already demonstrated that the group will not act as an individual if tastes are homothetic but different.

Exercise 15A.5 *Suppose that my wife and I share identical homothetic tastes (that are not over perfect substitutes). Will our household demand curve be identical to our marginal willingness to pay curve?*

Answer: No. When tastes are homothetic, they give rise to income effects — which implies that individual demand curves and marginal willingness to pay curves will differ, and this difference continues to hold when we consider the aggregate demand curve. Thus, even though we behave as a representative agent, our demand and marginal willingness to pay curves will not be the same.

Exercise 15A.6 *Does this measure of long run profit apply also when the firm encounters long run fixed costs?*

Answer: Yes. This is because the fixed cost is included in the long run AC and is therefore counted, and the presence of fixed costs does not change the *additional* cost incurred by producing more than the quantity at the lowest point of the AC curve.

Exercise 15A.7 *How would the picture be different if we were depicting an industry in long run equilibrium with all firms facing the same costs? What would long run producer surplus be in that case?*

Answer: The long run supply curve would then be flat — which would eliminate the producer surplus area entirely from the graph. This should make sense: In a competitive industry where all firms face the same costs, entry and exit drive long run profit to zero. Thus, while each firm will earn short run profits (because certain long run costs are not costs in the short run), the industry will earn zero profit in the long run. The entire surplus in the market would then be earned by consumers.

Exercise 15A.8 *Suppose we were not concerned about identifying producer and worker surplus but instead wanted to only predict the equilibrium wage and the number of workers employed. Would we then also have to assume that leisure is quasilinear for workers?*

Answer: No — in order to predict the market equilibrium, we simply need to know the aggregate demand and supply curves in the market. We can aggregate these even if consumers (or workers) do not behave as one single representative agent. Put differently, we need the regular consumer demand or worker supply curves to predict the equilibrium, not the compensated consumer demand and worker supply curves.

Exercise 15A.9 *Imagine that you are Barney and that you would like consumers to get a bigger share of the total "pie" than they would get in a decentralized market. How might you accomplish this? (*Hint*: Given your omnipotence, you are not restricted to charging the same price to everyone.)*

Answer: All you would have to do is charge a lower price to some of the consumers. You could still give enough to producers so that their surplus is positive — but you could then redistribute some of the surplus from producers to consumers. In the extreme, you would simply cover the costs of producers and hand all the goods to the consumers who value them most, charging them only a price sufficient to raise enough money for you to pay off the producers.

Exercise 15A.10 *Suppose the social marginal cost curve is perfectly flat — as it would be in the case of identical producers in the long run. Would you, as Barney, be able to give producers a share of the surplus?*

Answer: Sure. All you would have to do is charge the consumers who really value the goods a lot more than the long run equilibrium price that would emerge in the market. That long run price is sufficient to cover all the long run costs for producers — but lots of consumers are willing to pay more. Thus, if you raise this additional revenue from consumers, you can redistribute some of the consumer surplus to producers who would, in the competitive long run market, make zero surplus.

Exercise 15A.11 *How would Graph 15.8 look if good x were an inferior good for all consumers?*

Answer: In this case the aggregate $MWTP$ curve would be shallower than the market demand curve, causing the actual consumer surplus to be smaller than what we would infer from just looking at the market demand curve.

Exercise 15A.12 True or False*: If goods are normal, we will underestimate the consumer surplus if we measure it along the market demand curve, and if goods are inferior we will overestimate it.*

Answer: This is true. You can see it for normal goods in the graph in the text — where the $MWTP$ curves are steeper than demand curves. Similarly, $MWTP$ curves are shallower than demand curves in the case of inferior goods — which implies the actual consumer surplus is smaller than what we would measure along the market demand curve.

15B Solutions to Within-Chapter-Exercises for Part B

Exercise 15B.1 *Demonstrate that the conditions in equation (15.1) are satisfied for the demand functions in (15.2).*

Answer: Taking the first derivatives with respect to I^m, we get

$$\frac{\partial x_i^m}{\partial I^m} = b_i(p_1, p_2) = \frac{\partial x_i^n}{\partial I^n}. \tag{15.1}$$

Then, taking second derivatives, we get

$$\frac{\partial^2 x_i^m}{\partial (I^m)^2} = 0 = \frac{\partial^2 x_i^n}{\partial (I^n)^2}. \tag{15.2}$$

Exercise 15B.2 *Can you see why equation (15.2) represents the most general way of writing demands that satisfy the conditions in equation (15.1)?*

Answer: First, the only way the second derivatives can be zero is if income enters linearly and thus drops out when the first derivative is taken. Thus, we know that income can only enter as I multiplied by something that is not also a function of income — i.e. if income enters in the form $Ib(p_1, p_2)$ where the function b is at most a function of the prices (and not income). Second, the only way the first derivatives with respect to income can be the same across individuals is if the term following income is the same for both individuals — because that is the term that remains when we take the first derivative. Thus, the function b cannot be a individual specific — i.e. it cannot have an n or m superscript, but it can vary for goods — i.e. it can have an i subscript. Finally, other terms can enter the demand equations so long as they are not dependent on income — and thus do not affect the first derivative. Thus, we can have an a function that is not dependent on income but depends on prices — and that can vary across goods and individuals (since it drops out when we take the derivative with respect to income).

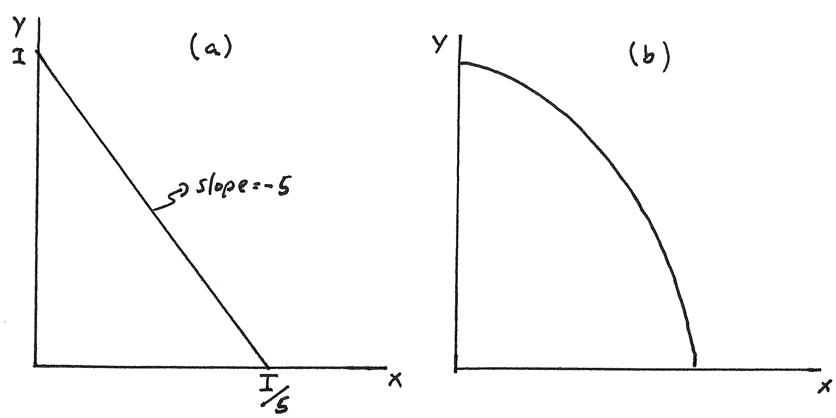
Exercise 15B.3 *What are my household demand functions (for x_1 and x_2 if my wife's and my individual demands are those in equation (15.3)? Does the household demand functions also satisfy the Gorman Form?*

Exercise 15B.4 *Given that the firms encounter a recurring fixed cost of \$1,280, which of the above functions should actually be qualified to take account of this fixed cost?*

Answer: The short run functions are not impacted, but the long run functions are. For instance, if $w = 20$ and $r = 10$, the lowest point of the AC function gives us a long run exit price of $p = 5$ — a price below which long run production falls to zero.

Exercise 15B.5 *Draw the production possibility frontier described above. How would it look differently if the long run market supply curve slopes up? (*Hint*: With an upward-sloping supply curve, society is facing an increasing cost of producing x, implying that the trade-off in the society-wide production possibility frontier must reflect that increasing cost.)*

Answer: In panel (a) of Graph 15.4, the production possibility frontier given by $I = 5x + y$ is given — with the frontier having slope -5 throughout because the (social) opportunity cost of increasing x by one unit is always that 5 units of y must be sacrificed. When the cost of producing x increases with the level of x (as it does when the supply curve is increasing), then we would get a production possibility frontier with the shape illustrated in panel (b) — where the slope starts shallow (indicating a low opportunity cost for producing x) but increases (in absolute value) as x increases (indicating the increasing opportunity cost.)



Graph 15.4: Production Possibility Frontiers

Exercise 15B.6 *Verify that this is indeed the case.*

Answer: The Lagrange function is

$$\mathcal{L} = 12,649.11x^{1/2} + y + \lambda(I - 5x - y) \tag{15.3}$$

which gives rise to the first order conditions

$$\frac{12,649.11}{2x^{1/2}} - 5\lambda = 0 \text{ and } 1-\lambda = 0. \tag{15.4}$$

Plugging the latter into the former and solving for x, we get $x = 1,600,000$.

Exercise 15B.7 *One way to verify that the representative consumer's utility function is truly "representative" is to calculate the implied demand curve and see whether it is equal to the aggregate demand curve $D^M(p) = 40,000,000/p^2$ that we are trying to represent. Illustrate that this is the case for the utility function $U(x,y) = 12,649.11x^{1/2} + y$.*

Answer: To derive the implied demand curve for the representative consumer, we solve the problem

$$\max_{x,y}\ 12,649.11x^{1/2} + y \text{ subject to } I = px + y. \tag{15.5}$$

Setting up the lagrange function

$$\mathcal{L} = 12,649.11x^{1/2} + y + \lambda(I - px - y), \tag{15.6}$$

we can derive the first order conditions

$$\frac{12,649.11}{2x^{1/2}} - \lambda p = 0 \text{ and } 1-\lambda = 0. \tag{15.7}$$

Substituting the latter into the former and solving for x, we get

$$x(p) = \frac{40,000,000}{p^2}, \tag{15.8}$$

precisely the aggregate demand function we are trying to represent with the representative consumer.

15C Solutions to End-of-Chapter Exercises

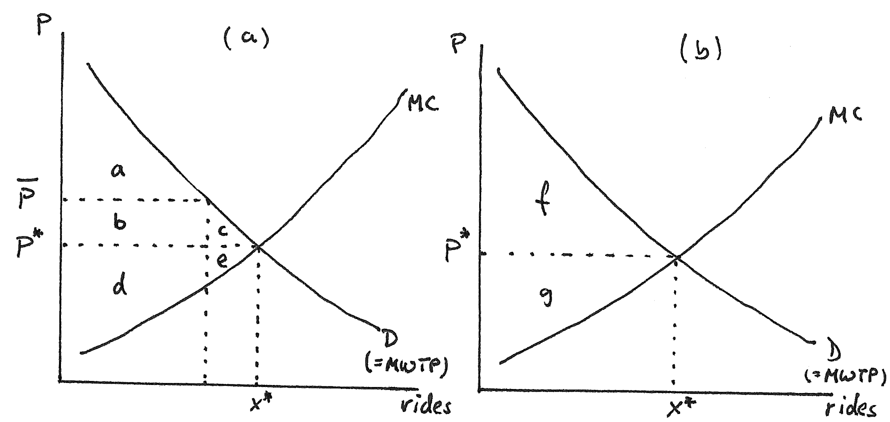
Exercise 15.2: *Disneyland Pricing Revisited*

Business Application: *Disneyland Pricing Revisited: In end-of-chapter exercise 33, we investigated different ways that you can price the use of amusement park rides in a place like Disneyland. We now return to this example. Assume throughout that consumers are never at a corner solution.*

A: *Suppose again that you own an amusement park and assume that you have the only such amusement park in the area — i.e. suppose that you face no competition. You have calculated your cost curves for operating the park, and it turns out that your marginal cost curve is upward sloping throughout. You have also estimated the downward sloping (uncompensated) demand curve for your amusement park rides, and you have concluded that consumer tastes appear to be identical for all consumers and quasilinear in amusement park rides.*

(a) *Illustrate the price you would charge per ride if your aim was to maximize the overall surplus that your park provides to society.*

Answer: This is illustrated in panel (a) of Graph 15.5 (next page) where demand intersects the MC curve at output x^* and price p^*. The price p^* maximizes the total surplus — because, for all quantities below x^*, the marginal benefit (measured by the $MWTP$ curve) exceeds the marginal cost of production. (The demand curve is equal to the $MWTP$ curve because of the quasilinearity of rides in consumer tastes.)



Graph 15.5: Amusement Park Rides

(b) *Now imagine that you were not concerned about social surplus and only about your own profit. Illustrate in your graph a price that is slightly higher than the one you indicated in part (a). Would your profit at that higher price be greater or less than it was in part (a)?*

Answer: This is illustrated in panel (a) using the price $\overline{p}$. At the original price p^*, profit is equal to $(d+e)$. At the new price $\overline{p}$, profit is equal to $(b+d)$. Since b is greater than e, profit is higher at the higher price. This is because, while you give up some profit from not producing as much, you more than make up for it by being able to sell what you do produce at a higher price.

(c) True or False*: In the absence of competition, you do not have an incentive to price amusement park rides in a way that maximizes social surplus.*

Answer: This is true (as already illustrated above). Note that social surplus falls from $(a+b+c+d+e)$ under p^* to $(a+b+d)$ under $\overline{p}$. Thus, while profit increases, social surplus falls because consumers lose more than the increase in profit.

(d) *Next, suppose that you decide to charge the per-ride price you determined in part (a) but, in addition, you want to charge an entrance fee into the park. Thus, your customers will now pay that fee to get into the park — and then they will pay the per-ride price for every ride they take. What is the most that you could collect in entrance fees without affecting the number of rides consumed?*

Answer: This is illustrated in panel (b) of Graph 15.5. At price p^*, consumers get a total consumer surplus of area f. That is the most they would be willing to pay to enter the park and face a price per ride of p^*. You could therefore charge a per-consumer entrance fee of area f divided by the number of consumers.

(e) *Will the customers that come to your park change their decision on how many rides they take? In what sense is the concept of "sunk cost" relevant here?*

Answer: No, once they pay the fee, they will continue to consume as many rides as before. In essence, once they pay the entrance fee, that fee is a sunk cost — and it does not affect decisions in the park. (This is technically true in this example only because of the quasilinearity of rides in consumer tastes. If rides were not quasilinear, then the entrance fee would alter the income available for consumption of rides and other goods — which would produce an income effect in addition to the substitution effect. But, since rides are quasilinear, there is no income effect.)

(f) *Suppose you collect the amount in entrance fees that you derived in part (d). Indicate in your graph the size of consumer surplus and profit assuming you face no fixed costs for running the park?*

Answer: By charging the maximum entrance fee derived in (d), you are in essence taking all the consumer surplus. Thus, in addition to earning the profit g from selling rides in the park, you are also earning the revenue f from the entrance fees — causing your total profit (in the absence of fixed costs) to be the area $(f+g)$.

(g) *If you do face a recurring fixed cost FC, how does your answer change?*

Answer: If you face recurring fixed costs FC, your (long run) profit will be $(f+g-FC)$.

(h) True or False*: The ability to charge an entrance fee in addition to per-ride prices restores efficiency that would be lost if you could only charge a per-ride price.*

Answer: This is true — total surplus will now be back to what it was in panel (a) of the Graph when price p^* was charged — except that the consumer surplus appears as profit instead of consumer surplus.

(i) *In the presence of fixed costs, might it be possible that you would shut down your park if you could not charge an entrance fee but you keep it open if you do?*

Answer: Yes. Suppose that recurring fixed costs were greater than the area g in panel (b) of the graph. Then you would earn profit if you could charge an entrance fee and possibly not if you cannot charge an entrance fee. (We don't know yet until we talk about monopolies later on in the text exactly how high fixed costs would have to be in order for you not to be able to operate without an entrance fee — because, as we illustrated earlier in the problem, you can in fact increase profit by charging a price higher than p^* if you cannot charge an entrance fee.)

B: *Suppose, as in exercise 33, tastes for your consumers can be modeled by the utility function $u(x_1,x_2) = 10x_1^{0.5}+x_2$, where x_1 represents amusement park rides and x_2 represents dollars of other consumption. Suppose further that your marginal cost function is given by $MC(x) = x/(250,000)$.*

(a) *Suppose that you have 10,000 consumers on any given day. Calculate the (aggregate) demand function for amusement park rides.*

Answer: Each consumer's demand function is calculated by solving the problem

$$\max_{x_1,x_2} 10x_1^{0.5}+x_2 \text{ subject to } I = px_1+x_2 \tag{15.9}$$

which gives the demand function $x_1(p) = 25/p^2$. Multiplying by 10,000, we get the aggregate demand function

$$X(p) = \frac{250,000}{p^2}. \tag{15.10}$$

(b) *What price would you charge if your goal was to maximize total surplus? How many rides would be consumed?*

Answer: Inverting the aggregate demand function gives us the aggregate demand curve $p = 500/(x^{0.5})$. Total surplus is maximized where this intersects the marginal cost curve. This implies that we have to solve the equation

$$\frac{500}{x^{0.5}} = \frac{x}{250,000} \tag{15.11}$$

which gives us $x = 250,000$. Plugging this back into the demand curve $p = 500/(x^{0.5})$, we get a per-ride price of $p = 1$ at which each consumer would demand 25 rides.

(c) *In the absence of fixed costs, what would your profit be at that price?*

Answer: At that price, your total revenues would be \$250,000. Your costs would be the area under the MC curve — which is \$125,000. (This is easy to calculate since the MC curve is linear and thus just half the total revenues. More generally, you would calculate this as the integral of the MC curve.) Thus, your profit (in the absence of fixed costs) is \$125,000.

(d) *Suppose you charged a price that was 25% higher. What would happen to your profit?*

Answer: If you charged a price of \$1.25 per ride (rather than the \$1 per ride you just calculated), the number of rides demanded would be

$$X(2) = \frac{250,000}{1.25^2} = 160,000. \tag{15.12}$$

This implies that your revenue would be $160,000(1.25) = \$200,000$. Your marginal cost at 160,000 would be $MC(160,000) = 160,000/250,000 = 0.64$. This implies that your total costs would be $0.64(160,000)/2) = \$51,200$ — which implies your profit would be $200,000 - 51,200 = \$148,800$. This is an increase in profit from the \$125,000 we calculated for the per-ride price of \$1.

(e) *Derive the expenditure function for your consumers.*

Answer: We first have to solve the expenditure minimization problem

$$\min_{x_1,x_2} px_1 + x_2 \text{ subject to } u = 10x_1^{0.5} + x_2 \tag{15.13}$$

which gives us the compensated demand functions

$$x_1(p) = \frac{25}{p^2} \text{ and } x_2(p,u) = u - \frac{50}{p}. \tag{15.14}$$

Using these, we can then derive the expenditure function

$$E(p,u) = p\left(\frac{25}{p^2}\right) + \left(u - \frac{50}{p}\right) = u - \frac{25}{p}. \tag{15.15}$$

(f) *Use this expenditure function to calculate how much consumers would be willing to pay to keep you from raising the price from what you calculated in (b) to 25% more. Can you use this to argue that raising the price by 25% is inefficient even though it raises your profit?*

Answer: The amount that each consumer is willing to pay to avoid this price increase is

$$E(1.25,\overline{u}) - E(1,\overline{u}) = \left(\overline{u} - \frac{25}{1.25}\right) - \left(\overline{u} - \frac{25}{1}\right) = \$5.00, \tag{15.16}$$

where we can leave the utility amount $\overline{u}$ unspecified since it cancels out. (You could also assume some sufficiently high income, calculate the utility the consumer gets at the initial price, and then us it to derive the amount the consumer is willing to pay to not incur the price increase. Your answer would be the same.)

Thus, each consumer is willing to pay \$5 to avoid the increase in per-ride prices from \$1 to \$1.25 — which implies that the 10,000 consumers are willing to pay \$50,000. Above, we calculated that profit increases from \$125,000 to \$148,800. Thus, the increase in price causes an increase of \$23,800 in profit but consumers would have been wiling to pay \$50,000 to avoid that increase — causing a social loss of \$26,200.

(g) *Next, determine the amount of an entrance fee that you could charge while continuing to charge the per-ride price you determined in (b) without changing how many rides are demanded.*

Answer: At a price of \$1 per ride, each consumer demands 25 rides. The total willingness to pay for these rides is the area under the demand curve $p = 5/x^{0.5}$; i.e.

$$\int_0^{25} \frac{5}{x^{0.5}} dx = 10x^{0.5}|_0^{25} = 50 - 0 = 50. \tag{15.17}$$

At a price of \$1 per ride, the consumer has to pay \$25 for the 25 rides she consumes once in the park — which means her consumer surplus is $50 - 25 = 25$. Thus, you could charge an entry fee of \$25.

(h) *How much is your profit now? What happens to consumer surplus? Is this efficient?*

Answer. Your profit now be the previously calculated profit of \$125,000 plus the revenue from the entrance fee which is $25(10,000) = \$250,000$ — for a total profit of \$375,000. There would now be no consumer surplus left. And yes, this would be efficient — because you are producing the efficient number of rides but simply transferred the consumer surplus to yourself.

(i) *Suppose the recurring fixed cost of operating the park is \$200,000. Would you operate it if you had to charge the efficient per-ride price but could not charge an entrance fee? What if you could charge an entrance fee?*

Answer: If you could not charge an entrance fee and had to charge the price of \$1 per ride, you would earn a profit before accounting for fixed costs of \$125,000. Once you take the fixed cost into account, you would therefore operate at a loss of \$75,000 and would not operate the park. However, if you could charge an entrance fee, you can earn up to \$375,000 in profit before accounting for fixed costs — which still leaves you a maximum profit of \$175,000 if you did charge the highest possible entrance fee. Thus, you would operate the park if you could also charge an entrance fee.

Exercise 15.5: *Redistribution of Income without Income Effects*

Policy Application: *Redistribution of Income without Income Effects*: *Consider the problem a society faces if it wants to both maximize efficiency while also insuring that the overall distribution of "happiness" in the society satisfies some notion of "equity".*

A: *Suppose that everyone in the economy has tastes over x and a composite good y, with all tastes quasilinear in x.*

(a) *Does the market demand curve (for x) in such an economy depend on how income is distributed among individuals (assuming no one ends up at a corner solution)?*

Answer: If everyone's tastes are quasilinear in x, this means that each person's demand for x is independent of income (unless someone is at a corner solution). Thus, the aggregate demand curve in the market for x does not depend on the distribution of income in the population. Since the supply curve also does not depend on the distribution of income, the market equilibrium in the x market is independent of the income distribution.

(b) *Suppose you are asked for advice by a government that has the dual objective of maximizing efficiency as well as insuring some notion of "equity". In particular, the government considers two possible proposals: Under proposal A, the government redistributes income from wealthier individuals to poorer individuals before allowing the market for x to operate. Under proposal B, on the other hand, the government allows the market for x to operate immediately and then redistributes money from wealthy to poorer individuals after equilibrium has been reached in the market. Which would you recommend?*

Answer: Since the market outcome in the x market is independent of the distribution of income, it does not matter whether income is redistributed before or after the market equilibrium has been reached. The end result will be exactly the same. Thus, you should tell the government it does not matter which policy is put in place.

(c) *Suppose next that the government has been replaced by an omniscient social planner who does not rely on market processes but who shares the previous government's dual objective. Would this planner choose a different output level for x than is chosen under proposal A or proposal B in part (b)?*

Answer: No, the social planner would do exactly what the government would do under either of the two policies. This is because the social planner is not restricting his ability to achieve different notions of equity by allowing surplus in the x market to be maximized — which happens when the competitive equilibrium quantity of x is produced.

(d) True or False*: As long as money can be easily transferred between individuals, there is no tension in this economy between achieving many different notions of "equity" and achieving efficiency in the market for x.*

Answer: This is true (as already explained in the previous part).

(e) *To add some additional realism to the exercise, suppose that the government has to use distortionary taxes in order to redistribute income between individuals. Is it still the case that there is no tradeoff between efficiency and different notions of equity?*

Answer: In this case, a tradeoff does emerge — because redistribution through distortionary taxes implies the creation of deadweight losses as income is transferred between individuals. Thus, more redistribution implies a loss of social surplus — thus the tension between "equity" and efficiency.

B: *Suppose there are two types of consumers: Consumer type 1 has utility function $u^1(x,y) = 50x^{1/2} + y$, and consumer type 2 has utility function $u^2(x,y) = 10x^{3/4} + y$. Suppose further that consumer type 1 has income of 800 and consumer type 2 has income of 1,200.*

(a) *Calculate the demand functions for x for each consumer type assuming the price of x is p and the price of y is 1.*

Answer: Using the utility function $u(x,y) = Ax^{\alpha} + y$, we can solve for the demand function for x as

$$2x(p) = \left(\frac{\alpha A}{p}\right)^{1/(1-\alpha)}. \tag{15.18}$$

Substituting for the terms in the two utility functions for the two types, this implies demand functions

$$x^1(p) = \left(\frac{0.5(50)}{p}\right)^{1/(1-0.5)} = \frac{625}{p^2} \text{ and } x^2(p) = \left(\frac{0.75(10)}{p}\right)^{1/(1-0.75)} = \frac{3,164.0625}{p^4} \tag{15.19}$$

for type 1 and 2 respectively.

(b) *Calculate the aggregate demand function when there are 32,000 of each consumer type.*

Answer: Multiplying each demand function by 32,000 and adding, we get

$$X(p) = \frac{32,000(625)}{p^2} + \frac{32,000(3,164.0625)}{p^4} = \frac{20,000,000p^2 + 101,250,000}{p^4}. \tag{15.20}$$

(c) *Suppose that the market for x is a perfectly competitive market with identical firms that attain zero long run profit when $p = 2.5$. Determine the long run equilibrium output level in this industry.*

Answer: Substituting $p = 2.5$ into the equation $X(p)$, we get $X(2.5) = 5,792,000$.

(d) *How much x does each consumer type consume?*

Answer: Consumer type 1 consumes $625/(2.5^2) = 100$ and consumer type 2 consumes $3,164.0625/(2.5^4) = 81$ units of x.

(e) *Suppose the government decides to redistribute income in such a way that, after the redistribution, all consumers have equal income — i.e. all consumers now have income of 1,000. Will the equilibrium in the x market change? Will the consumption of x by any consumer change?*

Answer: Income does not enter any demand function (because the good x is quasilinear) — which implies that the income distribution does not enter the aggregate demand function $X(p)$. Thus, redistributing income in this way does not change either the equilibrium level of output in the market or the level of x consumption of any individual.

(f) *Suppose instead of a competitive market, a social planner determined how much x and how much y every consumer consumes. Assume that the social planner is concerned about both the absolute welfare of each consumer as well as the distribution of welfare across consumers — with more equal distribution more desirable. Will the planner produce the same amount of x as the competitive market?*

Answer: Yes — social surplus is still maximized at the same output level regardless of how the planner decides to redistribute income (so long as no one ends up at a corner solution). Thus, the planner would want to maximize the surplus in the x market by picking the same output level as the market — and he can then worry about redistributing income to the desired level.

(g) True or False*: The social planner can achieve his desired outcome by allowing a competitive market in x to operate and then simply transferring y across individuals to achieve the desired distribution of happiness in society.*

Answer: This is true. In other words, in an economy where all tastes are quasilinear in x, the planner does not actually have to calculate the optimal quantity of x but can rather allow the market to determine that quantity since it is unaffected by how income is distributed. By shifting y from some people to others, the planner can then achieve whatever desired level of "equity" he desires.

(h) *Would anything in your analysis change if the market supply function were upward sloping?*

Answer: Since the market demand curve is unaffected by redistribution of income, the market demand would continue to intersect market supply at the same point regardless of whether or not the supply curve slopes up. Thus, nothing changes fundamentally in the problem if we assume an upward sloping supply curve.

(i) *Economists sometimes refer to economies in which all individuals have quasilinear tastes as "transferable utility economies" — which means that in economies like this, the government can transfer happiness from one person to another. Can you see why this is the case if we were using the utility functions as accurate measurements of happiness?*

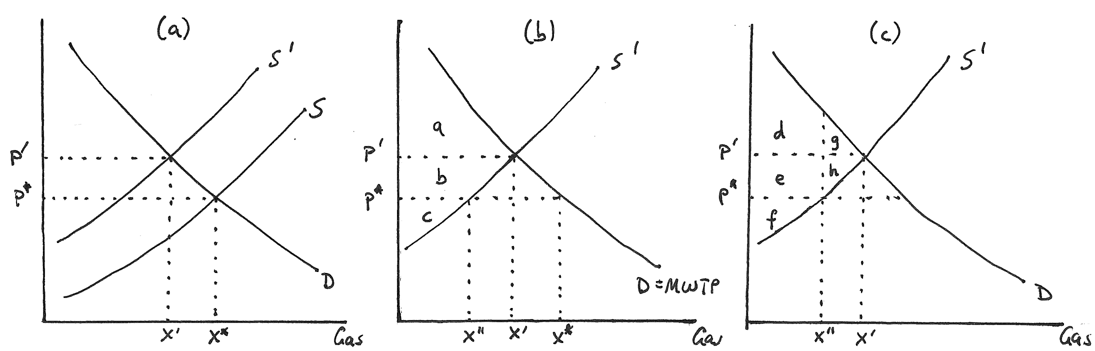
Answer: If we use the two utility functions in this problem as accurate measurements of happiness, then the planner will increase utility by 1 unit for a person of type 1 and lower it by 1 unit for a person of type 2 if he transfers one unit of y from person 1 to person 2. Thus, he is in essence able to transfer utility between individuals.

Exercise 15.9: *Anti-Price-Gouging Laws*

Policy Application: *Anti-Price Gauging Laws*: *As we will discuss in more detail in Chapter 18, governments often interfere in markets by placing restrictions on the price that firms can charge. One common example of this is so-called "anti-price gauging laws" that restrict profits for firms when sudden supply shocks hit particular markets.*

A: *A recent hurricane disrupted the supply of gasoline to gas stations on the East Coast of the U.S. Some states in this region enforce laws that prosecute gasoline stations for raising prices as a result of natural disaster-induced drops in the supply of gasoline.*

(a) *On a graph with weekly gallons of gasoline on the horizontal and price per gallon on the vertical, illustrate the result of a sudden leftward shift in the supply curve (in the absence of any laws governing prices.)*



Graph 15.6: Anti-Price Gauging Laws

Answer: This is illustrated in panel (a) of Graph 15.6 (previous page) where S is the original supply curve and S' is the new supply curve. The equilibrium shifts from one where price was p^* and gasoline consumption x^* to one where the price is p' and gasoline consumption is x'.

(b) *Suppose that gasoline is a quasilinear good for consumers. Draw a graph similar to the one in part (a) but include only the post-hurricane supply curve (as well as the unchanged demand curve). Illustrate consumer surplus and producer profit if price is allowed to settle to its equilibrium level.*

Answer: This is illustrated in panel (b) of Graph 15.6. Consumer surplus would be equal to area a and producer profit would be equal to area $(b+c)$.

(c) *Now consider a state that prohibits price adjustments as a result of natural disaster-induced supply shocks. How much gasoline will be supplied in this state? How much will be demanded?*

Answer: This is also illustrated in panel (b). At the pre-crisis price of p^*, firms would supply x'' — but consumers would want to buy x^*.

(d) *Suppose that the limited amount of gasoline is allocated at the pre-crisis price to those who are willing to pay the most for it. Illustrate the consumer surplus and producer profit.*

Answer: This is illustrated in panel (c) of Graph 15.6. If the limited amount of gasoline x'' is bought at p^* by those who value it the most, then consumer surplus is $(d+e)$. Producer profit is area f.

(e) *On a separate graph, illustrate the total surplus achieved by a social planner who insures that gasoline is given to those who value it the most and sets the quantity of gasoline at the same level as that traded in part (c). Is the social surplus different than what arises under the scenario in (d)?*

Answer: The social surplus would then be the same as in part (d) — equal to area $(d+e+f)$.

(f) *Suppose that instead the social planner allocates the socially optimal amount of gasoline. How much greater is social surplus?*

Answer: The socially optimal quantity is x'. If that much is produced, the total surplus is $(d+e+f+g+h)$ — which is greater than the surplus under the restricted quantity x'' by area $(g+h)$.

(g) *How does the total social surplus in (f) compare to what you concluded in (b) that the market would attain in the absence of anti-price gauging laws?*

Answer: It is identical.

(h) True or False*: By interfering with the price signal that communicates information about where gasoline is most needed, anti-price gauging laws have the effect of restricting the inflow of gasoline to areas that most need gasoline during times of supply disruptions.*

Answer: This is true, as demonstrated in the problem. The areas where gasoline would be most needed are those where the price would rise most in the absence of anti-price gauging laws. Thus, it is in these areas that the greatest shortages would emerge.

B: *Suppose again that the aggregate demand function $X^D(p) = 250{,}000/p^2$ arises from 10,000 local consumers of gasoline with quasilinear tastes (as in exercise* **??***).*

(a) *Suppose that the industry is in long run equilibrium — and that the short run industry supply function in this long run equilibrium is $X^S(p) = 3{,}906.25p$. Calculate the equilibrium level of (weekly) local gasoline consumption and the price per dollar.*

Answer: Setting $X^D(p) = X^S(p)$, we get $p = 4$. Substituting this back into either the demand or supply equation, we get $x = 15{,}625$.

(b) *What is the size of the consumer surplus and (short run) profit?*

Answer: The consumer surplus is

$$\int_4^{\infty} \frac{250{,}000}{p^2}\,dp = -\frac{250{,}000}{p}\Big|_4^{\infty} = 0-(-62{,}500) = \$62{,}500. \qquad (15.21)$$

The firm (short run) profits are

$$\int_0^4 3,906.25p\,dp = 1,953.125p^2|_0^4 = 31,250 - 0 = \$31,250. \tag{15.22}$$

(c) *Next suppose that the hurricane-induced shift in supply moves the short run supply function to* $\overline{X}^S = 2,000p$. *Calculate the new (short run) equilibrium price and output level.*

Answer: We solve for the new equilibrium price by setting $X^D(p) = \overline{X}^S(p)$ and solving for $p = 5$. Plugging this back into either the demand or supply functions, we get $x = 10,000$.

(d) *What is the sum of consumer surplus and (short run) profit if the market is allowed to adjust to the new short run equilibrium?*

Answer: Consumer surplus is now

$$\int_5^\infty \frac{250,000}{p^2}dp = -\frac{250,000}{p}|_5^\infty = 0 - (-50,000) = \$50,000. \tag{15.23}$$

Profits for firms are

$$\int_0^5 2,000p\,dp = 1,000p^2|_0^5 = 25,000 - 0 = \$25,000. \tag{15.24}$$

Thus, the sum of consumer surplus and (short run) firm profits is \$75,000.

(e) *Now suppose the state government does not permit the price of gasoline to rise above what you calculated in part (a). How much gasoline will be supplied?*

Answer: At a price of $p = 4$, the gallons of gasoline supplied will be

$$\overline{X}^S(4) = 2,000(4) = 8,000. \tag{15.25}$$

(f) *Assuming that the limited supply of gasoline is bought by those who value it the most, calculate overall surplus (i.e. consumer surplus and (short run) profit) under this policy.*

Answer: The easiest way to calculate this is to find the area under the demand *curve* that lies above the supply curve up to $x = 8,000$. The area under the demand curve is

$$\int_0^{8000} \frac{500}{x^{0.5}}dx = 1,000x^{0.5}|_0^{8000} \approx \$89,442.72. \tag{15.26}$$

The supply *curve* is the supply function solved for p — i.e. $p = 0.0005x$. The area under the supply curve up to $x = 8000$ is

$$\int_0^{8000} 0.0005x\,dx = 0.00025x^2|_0^{8000} = \$16,000. \tag{15.27}$$

Thus, the overall surplus is $89442.72 - 16000 = \$73,442.72$.

(g) *How much surplus is lost as a result of the government policy to not permit price increases in times of disaster-induced supply shocks?*

Answer: In the absence of the policy, total surplus was \$75,000 — which is \$1,557.28 greater than the total surplus under the policy.

Conclusion: Potentially Helpful Reminders

1. The idea of representing different sides of the market as if they emerged from the behavior of a “representative agent” is a powerful one because it allows us to treat certain market curves using our insights from the development of consumer and producer theory.

2. It is only when a market relationship emerges from consumer theory — as it does in the case of consumer demand and labor supply — that we have to be careful as we are tempted to think about these market relationships as if they emerged from a single optimization problem. This is because of the presence of income effects that, when assumed away, remove the difficulty. It is for this reason that the analysis of welfare becomes significantly more straightforward when we assume quasilinear tastes.

3. Remember that you can always use market relationships to predict market outcomes — regardless of whether the condition of quasilinearity holds. It is only when we then try to determine exact welfare measures that we have to be careful if quasilinearity does not hold — which implies that it is only when quasilinearity does not hold that we have to worry about separately thinking about compensated rather than uncompensated relationships.

4. The complications of introducing income effects are explored in a particularly revealing way in end-of-chapter exercises 15.5 and 15.6 where we show how redistribution of income does not alter the equilibrium in a market under some conditions (quasilinearity) but does do so under other conditions (i.e. when there are income effects). Nevertheless, we show that the market equilibrium will retain its efficiency property under the assumptions of the first welfare theorem even in the presence of income effects — even though the nature of the equilibrium will depend on the initial distribution of income in that case.

5. It is important from the outset to be aware of the limitations of the first welfare theorem — limitations that arise from the underlying assumptions listed in the chapter and developed throughout the remainder of the text. End-of-chapter exercises 15.2, 15.7, 15.8 and 15.9 begin to explore these.