CHAPTER

# 4

# Tastes and Indifference Curves

This chapter begins a 2-chapter treatment of tastes — or what we also call "preferences". In the first of these chapters, we simply investigate the basic logic behind modeling tastes and the most fundamental assumptions we make. In the next chapter, we then turn to what specific types of tastes look like. Call me geek — but I think it's pretty cool that we have found ways of systematically modeling something as abstract as people's tastes!

## Chapter Highlights

The main points of the chapter are:

1. **Tastes have nothing to do with budgets** — they are conceptually distinct. Budgets are all about what is feasible — and they arise objectively from "what we bring to the table" and the prices we face. Tastes are all about what we desire and have nothing to do with incomes, endowments or prices.

2. While our model of tastes respects the fact that there is a **great diversity of tastes across people**, we assume that **some aspects of tastes are constant across individuals**. The most basic of these are completeness and transitivity, but the assumptions of monotonicity, convexity and continuity are also intuitively appealing in most circumstances. When we say tastes are "rational" we only mean that individuals with those tastes are capable of making decisions.

3. **Maps of indifference curves are a way of describing tastes**, with the usual shapes and orderings arising from our assumptions of monotonicity and convexity. For those covering part B, indifference curves are simply levels of utility functions — and these levels can no more cross than the elevations of mountains on geography maps can cross.

4. We typically assume that **utility cannot be measured objectively** — which is why the labels on indifference curves do not matter beyond indicating the **ordering** of what is better and what is worse.

5. The slope of an indifference curve — or the **marginal rate of substitution** — at a particular bundle tells us how an individual feels about different goods *at the margin* — i.e. how much the individual is willing to trade one good for another *given that she currently has this particular bundle*.

# Using the LiveGraphs

For an overview of what is contained on the LiveGraphs site for each of the chapters (from Chapter 2 through 29) and how you might utilize this resource, see pages 2-3 of Chapter 1 of this *Study Guide*. To access the LiveGraphs for Chapter 4, click the *Chapter 4* tab on the left side of the LiveGraphs web site.

If you are not covering the mathematical B-part of the text *Microeconomics: An Intuitive Approach with Calculus* (or if you are using the non-mathematical text entitled *Microeconomics: An Intuitive Approach*), you will miss some very helpful LiveGraphs unless you look for them. You can either find these under *Animated Graphics* if you are on the LiveGraphs site for the larger book with calculus — or you can find them under the *Exploring Relationships* tab if you are on the LiveGraphs site for the shorter intuitive text without calculus. The four animations include:

1. First, the animation for *Indifference Curves & Utility Functions: 3D Object in 2D Space* (which is Graph 4.8 in the calculus-based text) illustrates how indifference curves relate to utility "mountains" (or utility functions).

2. Second, the two animations referring to "Mount Nechyba" (which appear as Graphs 4.9 and 4.10 in the calculus-based text) illustrate how we graph 3D mountains in two dimensions — exactly as we do indifference curves.

3. Third, a final graph re-scales the utility function (or "mountain") and illustrates how only the labels but not the shapes of indifference curves change as a result. (This is Graph 4.11 in the calculus-based text.)

# 4A Solutions to Within-Chapter-Exercises for Part A

**Exercise 4A.1** *Do we know from the monotonicity assumption how $E$ relates to $D$, $A$ and $B$? Do we know how $A$ relates to $D$?*

Answer: $E$ must be preferred to $D$ because it contains more of everything (i.e. more pants and more shirts). Monotonicity does not tell us anything about the relationship between $A$ and $E$ — $A$ has more shirts but fewer pants and $E$ has more pants but fewer shirts. For analogous reasons, monotonicity does not tell us anything about how $E$ and $B$ are ranked. $A$ has more shirts and the same number of pants as $D$ — so we know that $A$ is at least as good as $D$ (and probably better).

**Exercise 4A.2** *What other goods are such that we would prefer to have fewer of them rather than many? How can we re-conceptualize choices over such goods so that it becomes reasonable to assume "more is better"?*

Answer: Examples might include pollution, bugs in our houses, weeds in our yard and disease in our bodies. In each case, however, we can re-conceptualize the "bad" by redefining it into a "good" that we want more of. We want less pollution but more clean air and water; fewer bugs in our houses or more "bug-free" square feet of housing; fewer weeds in our yard but more square feet of weed-less grass; less disease and more health.

**Exercise 4A.3** *Combining the convexity and monotonicity assumptions, can you now conclude something about the relationship between the pairs $E$ and $A$ and $E$ and $B$ if you do not know how $A$ and $B$ are related? What if you know that I am indifferent between $A$ and $B$?*

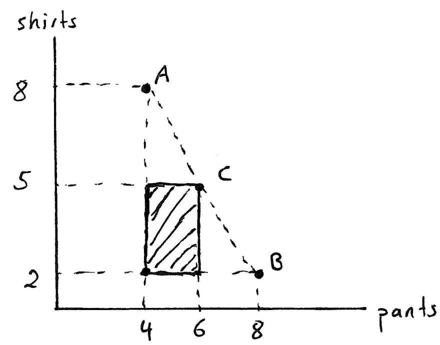
Answer: We can only apply the convexity assumption if we know some pair of bundles we are indifferent between — because convexity says that, when faced with bundles we are indifferent between, we prefer averages of such bundles (or at the very least like averages just as much). So, without knowing more, I can't use monotonicity and convexity to say anything about how $A$ and $E$ (or $B$ and $E$) are related to one another. If we know that I am indifferent between $A$ and $B$, on the other hand, then I know that $C$ is at least as good as $A$ and $B$ because $C$ is the average between $A$ and $B$. Since $E$ has more of everything than $C$, we also know from monotonicity that $E$ is better than $C$. So $E$ is better than $C$ which is at least as good as $A$ and $B$. By transitivity, that implies that $E$ is better than $A$ and $B$.

**Exercise 4A.4** *Knowing that I am indifferent between $A$ and $B$, can you now conclude something about how $B$ and $D$ are ranked by me? In order to reach this conclusion, do you have to invoke the convexity assumption?*

Answer: By just invoking the monotonicity assumption, I know that $A$ is at least as good as $D$ since it has more of one good and the same of the other. If $A$ is indifferent to $B$, I then also know (by transitivity) that $B$ is at least as good as $D$. Invoking convexity won't actually allow me to say anything beyond that since indifference between $A$, $B$ and $D$ is consistent with convexity. (It is not consistent with a *strict* notion of convexity — where by "strict" we mean that averages are strictly better than (indifferent) extremes. In that case, $A$ and $B$ are definitely preferred to $D$ if we are indifferent between $A$ and $B$.)

**Exercise 4A.5** *Illustrate the area in Graph 4.2b in which $F$ must lie — keeping in mind the monotonicity assumption.*

Answer: By monotonicity, $F$ must have less than $C$ and must therefore lie to the southwest of $C$. Thus, it must have no more than 5 shirts and no more than 6 pants. But it also cannot have fewer than 4 pants because then it would contain fewer pants and shirts than $A$ and would therefore be worse than $A$. And it cannot have fewer than 2 shirts because it would then have less of everything than $B$ and could no longer be indifferent to $B$. $F$ must therefore lie in the area illustrated in Graph 4.1.



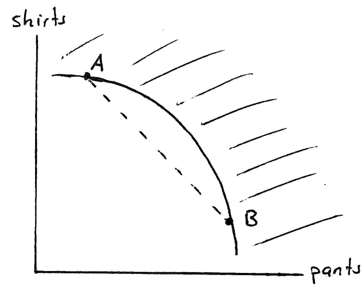

Graph 4.1: Graph for Within-Chapter-Exercise 4A.5

**Exercise 4A.6** *Suppose our tastes satisfy* weak *convexity in the sense that averages are just as good (rather than strictly better than) extremes. Where does F lie in relation to C in that case?*

Answer: In that case $F$ is the same bundle as $C$ — because $C$ is the average of the more extreme bundles $A$ and $B$.

**Exercise 4A.7** *Suppose extremes are better than averages. What would an indifference curve look like? Would it still imply diminishing marginal rates of substitution?*

Answer: The indifference curve would bend away from instead of toward the origin, as illustrated in Graph 4.2 (on the next page). The shaded area to the north-east of the indifference curve would contain all the better bundles (because of monotonicity). But the line connecting $A$ and $B$ — which contains averages between $A$ and $B$ — does not lie in this "better" region. Therefore, averages are worse than extremes. The slope of this indifference curve is then shallow at $A$ and becomes steeper as we move along the indifference curve to $B$. Thus, the marginal rate of substitution is no longer diminishing along the indifference curve — and the indifference curve exhibits increasing marginal rates of substitution.




Graph 4.2: Non-concex tastes

**Exercise 4A.8** *Suppose averages are just as good as extremes? Would it still imply diminishing marginal rates of substitution?*

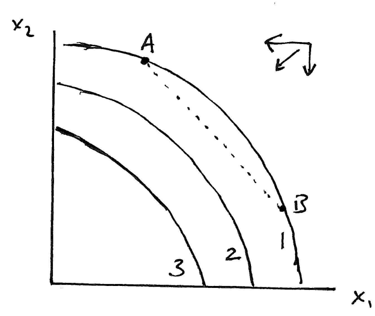
Answer: If averages are just as good as extremes, then indifference curves are straight lines. As a result, the slope would be the same along an indifference curve — implying constant rather than diminishing marginal rates of substitution. This is the borderline case between strictly convex tastes that have diminishing marginal rates of substitution and strictly non-convex tastes that have strictly increasing marginal rates of substitution.

**Exercise 4A.9** *Show how you can prove the last sentence in the previous paragraph by appealing to the transitivity of tastes.*

Answer: Pick any bundle that lies on the bold portion of the indifference curve to the southwest of $E$ and call it $B$. As noted in the text, we know from monotonicity that $E$ is better than $B$. Because $A$ and $B$ lie on the same indifference curve, you are indifferent between them. Thus, $E$ is better than $B$ which is indifferent to $A$. Transitivity then implies that $E$ is better than $A$.

**Exercise 4A.10** *Suppose less is better than more and averages are better than extremes. Draw three indifference curves (with numerical labels) that would be consistent with this.*

Answer: Graph 4.3 (on the next page) illustrates three such curves. Since less is better, the consumer becomes better off in the direction of the arrows at the top right of the graph. Thus, if I take $A$ and $B$ that lie on the same indifference curve, the line connecting them (which contains averages of the two) lies fully in the region that is more preferred. Thus, averages are indeed better than extremes. Since the consumer becomes better off as she moves southwest, the numbers accompanying the indifference curves must be increasing as we approach the origin.




Graph 4.3: Convex tastes over "bads"

## 4B Solutions to Within-Chapter-Exercises in Part B

**Exercise 4B.1** *True or False: If only one of the statements in (4.6) is true for a given set of bundles, then that statement's "$\succsim$" can be replaced by "$\succ$".*

Answer: True. If both statements are true, then the consumer is indifferent between the $A$ and the $B$ bundles (because that is the only way that the $A$ bundle can be at least as good as $B$ and the $B$ bundle can be at least as good as $A$ at the same time). If only one of the statements is true, then the consumer is not indifferent between the bundles. That must mean that one of the bundles is strictly preferred to the other, which means we can indeed replace "$\succsim$" with "$\succ$".

**Exercise 4B.2** *Does transitivity also imply that (4.8) implies (4.9) when "$\succsim$" is replaced by "$\succ$"?*

Answer: Yes. If $A$ is strictly preferred to $B$ and $B$ is strictly preferred to $C$, transitivity implies that $A$ must be strictly preferred to $C$.

**Exercise 4B.3** *True or False: Assuming tastes are transitive, the third line in expression (4.11) is logically implied by the first and second lines.*

Answer: True. Suppose we call the averaged bundle $C$. Then the first two lines say that the consumer being indifferent between $A$ and $B$ implies that she thinks $C$ is at least as good as $A$. Thus, $C \succsim A \sim B$ implies by transitivity that $C \succsim B$, which is what the third line says.

**Exercise 4B.4** *If you were searching for the steepest possible straight route up the last 2,000 feet of Mount "Nechyba" (in Graph 4.9), from what direction would you approach the mountain?*

Answer: It looks like you would approach it from the northwest (heading up the mountain toward the southeast) — because that is where the levels in the graph are closest to one another (which is where the mountain must be steepest).

**Exercise 4B.5** *In Political Science models, politicians are sometimes assumed to choose between bundles of spending on various issues — say military and domestic spending. Since they have to impose taxes to fund this spending, more is not necessarily better than less, and thus most politicians have some ideal bundle of domestic and military spending. How would such tastes be similar to the geographic mountain analogy?*

Answer: Such tastes would be similar in that the "utility mountain" would have a peak just like geographic mountains do. This is not usually the case for our "utility mountains" because usually we make the assumption that more is better — which means we can always climb higher up a mountain without peak. (More on this in end-of-chapter exercise 4.11.)

**Exercise 4B.6** *How does the expression for the marginal rate of substitution change if tastes could instead be summarized by the utility function* $u(x_1,x_2)=x_1^{1/4}x_2^{3/4}$

Answer: We would calculate this as

$$MRS = -\frac{(1/4)(x_1^{-3/4}x_2^{3/4})}{(3/4)(x_1^{1/4}x_2^{-1/4})} = -\frac{x_2}{3x_1}. \tag{4.1}$$

**Exercise 4B.7** *Can you verify that squaring the utility function in exercise 4B.6 also does not change the underlying indifference curves?*

Answer: Squaring the utility function from the previous exercise results in $v(x_1,x_2) = (u(x_1,x_2))^2 = \left(x_1^{1/4}x_2^{3/4}\right)^2 = x_1^{1/2}x_2^{3/2}$. This will give rise to the same indifference curves so long as the $MRS$ everywhere remains unchanged. The $MRS$ is

$$MRS = -\frac{(1/2)(x_1^{-1/2}x_2^{3/2})}{(3/2)(x_1^{1/2}x_2^{1/2})} = -\frac{x_2}{3x_1}, \tag{4.2}$$

exactly as it was before. Thus, the shape of the indifference curves is unaffected.

**Exercise 4B.8** *Illustrate that the same conclusion we reached with respect to* $u$ *and* $v$ *representing the same indifference curves also holds when we take the square root of* $u$ *— i.e. when we consider the function* $w(x_1,x_2)=(x_1^{1/2}x_2^{1/2})^{1/2}=x_1^{1/4}x_2^{1/4}$.

Answer: The $MRS$ is then

$$MRS = -\frac{(1/4)(x_1^{-3/4}x_2^{1/4})}{(1/4)(x_1^{1/4}x_2^{-3/4})} = -\frac{x_2}{x_1}, \tag{4.3}$$

exactly as it was when we calculated the $MRS$ for $u(x_1,x_2)=x_1^{1/2}x_2^{1/2}$ in the text.

**Exercise 4B.9** *Consider the utility function $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$. Take natural logs of this function and calculate the MRS of the new function. Is the natural log transformation one that can be applied to utility functions such that the new utility function represents the same underlying tastes?*

Answer: Taking logs, we get: $\ln u(x_1, x_2) = \ln\left(x_1^{1/2} x_2^{1/2}\right) = (1/2)\ln x_1 + (1/2)\ln x_2$. Note that the derivative of this with respect to $x_1$ is $1/(2x_1)$ and the derivative with respect to $x_2$ is $1/(2x_2)$. The $MRS$ is then

$$MRS = -\frac{1/(2x_1)}{1/(2x_2)} = -\frac{x_2}{x_1}, \tag{4.4}$$

exactly as it was before the log transformation. Thus, taking logs does not change the shape of indifference curves. Logs also do not change the ordering of the labels on indifference curves. Thus, when we take the log of a utility function, the new utility function represents the same tastes.

**Exercise 4B.10** *Consider the utility function $u(x_1, x_2, x_3) = x_1^{1/2} x_2^{1/2} x_3^{1/2}$. Take natural logs of this function and calculate the marginal rates of substitution of each pair of goods. Is the natural log transformation one that can be applied to utility functions of three goods such that the new utility function represents the same underlying tastes?*

Answer: Taking logs, we get a new function $v(x_1, x_2, x_3) = (1/2)\ln x_1 + (1/2)\ln x_2 + (1/2)\ln x_3$. Taking any pair of good $x_i$ and $x_j$ (where $i$, and $j$ can take values of 1, 2, and 3 but $i \neq j$), we get

$$MRS = -\frac{1/(2x_i)}{1/(2x_j)} = -\frac{x_j}{x_i}. \tag{4.5}$$

If we instead work with the original utility function $u(x_1, x_2, x_3) = x_1^{1/2} x_2^{1/2} x_3^{1/2}$, we can similarly calculate the $MRS$ between $x_i$ and $x_j$ while denoting the third good as $x_k$:

$$MRS = -\frac{(1/2)x_i^{-1/2} x_j^{1/2} x_k^{1/2}}{(1/2)x_i^{1/2} x_j^{-1/2} x_k^{1/2}} = -\frac{x_j}{x_i}. \tag{4.6}$$

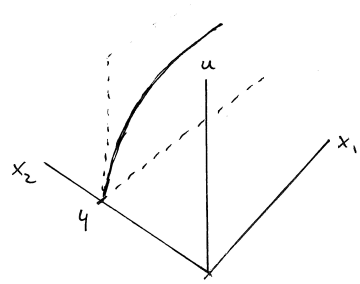
We therefore again get the same expressions for the $MRS$ between any two goods after we take logs of the utility function as we do before. Logs are general transformations that can always be applied to a utility function (regardless of how many goods the function is over) to get a new utility function that represents the same underlying tastes.

**Exercise 4B.11** *What would be the expression of the slope of the slice of the utility function $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$ when $x_1$ is fixed at 9? What is the slope of that slice when $x_2 = 4$?*

Answer: When $x_1 = 9$, the expression reduces to $(1/2)(9)^{1/2} x_2^{-1/2} = (3/2)x_2^{-1/2}$. This is the expression of the slope of the slice holding $x_1 = 9$. When $x_2 = 4$, that slope is $(3/2)(4)^{-1/2} = 3/4$.

**Exercise 4B.12** *Calculate $\partial u/\partial x_1$ for $u(x_1,x_2) = x_1^{1/2}x_2^{1/2}$. What does this reduce to when $x_2$ is fixed at 4? Where in Graph 4.12 does the slice along which this partial derivative represents the slope lie?*

Answer: $\partial u/\partial x_1 = (1/2)x_1^{-1/2}x_2^{1/2}$ reduces to $x_1^{-1/2}$ when $x_2 = 4$. The relevant slice is depicted in Graph 4.4.




Graph 4.4: Slice holding $x_2$ constant at 4

**Exercise 4B.13** *Calculate $\partial u/\partial x_1$ for the function $u(x_1,x_2) = 10\ln x_1 + 5\ln x_2$.*

Answer: $\partial u/\partial x_1 = 10/x_1$.

**Exercise 4B.14** *Calculate $\partial u/\partial x_1$ for the function $u(x_1,x_2) = (2x_1+3x_2)^3$. (Remember to use the Chain Rule.)*

Answer: $\partial u/\partial x_1 = 3(2x_1+3x_2)^2(2) = 6(2x_1+3x_2)^2$.

**Exercise 4B.15** *Verify that equation (4.28) is correct.*

Answer: The partial derivatives are

$$\frac{\partial u}{\partial x_1} = \frac{1}{2}x_1^{-1/2}x_2^{1/2} = \frac{x_2^{1/2}}{2x_1^{1/2}} \tag{4.7}$$

and

$$\frac{\partial u}{\partial x_2} = \frac{1}{2}x_1^{1/2}x_2^{-1/2} = \frac{x_1^{1/2}}{2x_2^{1/2}}. \tag{4.8}$$

When substituted into the equation, it verifies what is in the text.

**Exercise 4B.16** *Calculate the total differential $du$ of $u(x_1,x_2) = 10\ln x_1 + 5\ln x_2$.*

Answer: This is

$$du = \frac{10}{x_1}dx_1 + \frac{5}{x_2}dx_2. \tag{4.9}$$

# End of Chapter Exercises

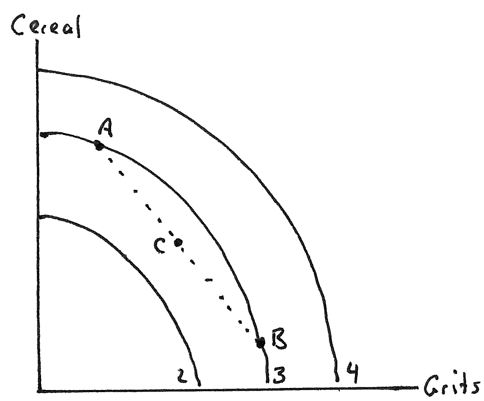
## Exercise 4.2

*Consider my wife's tastes for grits and cereal.*

**A:** *Unlike me, my wife likes both grits and cereal, but for her, averages (between equally preferred bundles) are worse than extremes.*

(a) *On a graph with boxes of grits on the horizontal and boxes of cereal on the vertical, illustrate three indifference curves that would be consistent with my description of my wife's tastes.*

Answer: This is illustrated in Graph 4.5. The shapes of these indifference curves arise from the following observation: Consider $A$ and $B$ that lie on the same indifference curve. Bundle $C$ is the average of $A$ and $B$ — and since averages are worse than extremes, $C$ must lie below the indifference curve that contains $A$ and $B$.




Graph 4.5: Grits and Cereal Tastes

(b) *Suppose we ignored labels on indifference curves and simply looked at shapes of the curves that make up our indifference map. Could my indifference map look the same as my wife's if I hate both cereal and grits? If so, would my tastes be convex?*

Answer: Yes, I would simply become better off as I move in the direction of the origin in the graph while my wife would become better off moving in the opposite direction. And my tastes would indeed be convex in this case — because $C$ would now lie "above" the indifference curve that contains $A$ and $B$ in the sense that I become better off as I move to the southwest in the graph.

**B:** *Consider the utility function $u(x_1, x_2) = x_1^2 + 4x_2^2$.*

(a) *Could this utility function represent the tastes you graphed in part A(a) above?*

Answer: Yes. To see this, we can calculate the marginal rate of substitution as

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{2x_1}{8x_2} = -\frac{x_1}{4x_2}. \tag{4.10}$$

When $x_1$ is low and $x_2$ is high, this implies that the $MRS$ is small in absolute value — and when $x_1$ is high and $x_2$ is low, it implies that the $MRS$ is large in absolute value. Put differently, for bundles that lie close to the vertical axis, the slope of the indifference curve is shallow, and for bundles that lie close to the horizontal axis, the slope of the indifference curve is steep. Thus, indifference curves exhibit increasing $MRS$ as we move from left to right — giving the shape in Graph 4.5.

(b) *How could you transform this utility function to be consistent with my tastes as described in A(b)?*

<u>Answer</u>: My tastes as described in A(b) give rise to the same indifference curves as my wife's, but I become better off as I get less of each good while she gets better off as she gets more. The function $u(x_1,x_2) = x_1^2 + 4x_2^2$ is increasing in both $x_1$ and $x_2$ — so it represents my wife's tastes. If I simply multiply it by negative 1, I get $v(x_1,x_2) = -x_1^2 - 4x_2^2$, a function that is decreasing in both $x_1$ and $x_2$. And the $MRS$ for the function $v$ is identical to the $MRS$ of the function $u$ — which implies the indifference curves maintain the same shape. We have thus transformed $u$ to $v$ where $u$ can represent my wife's tastes and $v$ can represent mine — with both of us having the same shapes of indifference curves.

## Exercise 4.5

*In this exercise, we explore the concept of marginal rates of substitution (and, in part B, its relation to utility functions) further.*

**A:** *Suppose I own 3 bananas and 6 apples, and you own 5 bananas and 10 apples.*

(a) *With bananas on the horizontal axis and apples on the vertical, the slope of my indifference curve at my current bundle is $-2$, and the slope of your indifference curve through your current bundle is $-1$. Assume that our tastes satisfy our usual five assumptions. Can you suggest a trade to me that would make both of us better off? (Feel free to assume we can trade fractions of apples and bananas).*

<u>Answer</u>: The slope of my indifference curve at my bundle tells us that I am willing to trade as many as 2 apples to get one more banana. The slope of your indifference curve at your bundle tells us that you are willing to trade apples and bananas one for one. If you offer me 1 banana in exchange for 1.5 apples, you would be better off because you would have been willing to accept as little as 1 apple for 1 banana. I would also be better off because I would be willing to give you as many as 2 apples for 1 banana — only having to give you 1.5 apples is better than that. (If you are uncomfortable with fractions of apples being traded, you could also propose giving me 2 bananas for 3 apples.)

This is only one possible example of a trade that would make us both better off. You could propose to give me 1 banana for $x$ apples, where $x$ can lie between 1 and 2. Since I am willing to give up as many as 2 apples for one banana, any such trade would make me better off, and since you are willing to trade them one for one, the same would be true for you.

(b) *After we engage in the trade you suggested, will our $MRS$'s have gone up or down (in absolute value)?*

<u>Answer</u>: Any trade that makes both of us better off moves me in the direction of more bananas and fewer apples — which, given diminishing marginal rates of substitution, should decrease the absolute value of my $MRS$; i.e. as I get more bananas and fewer apples, I will be willing to trade fewer apples to get one more banana than I was willing to originally. You, on the other hand, are giving up bananas and getting apples, which moves you in the opposite direction toward fewer bananas and more apples. Thus, you will become less willing to grade 1 banana for 1 apple and will in future trades demand more bananas in exchange for 1 apple. Thus, in absolute value, your $MRS$ will get larger.

(c) *If the values for our $MRS$'s at our current consumption bundles were reversed, how would your answers to (a) and (b) change?*

Answer: The trades would simply go in the other direction; i.e. I would be willing to trade 1 banana for $x$ apples so long as $x$ is at least 1, and you would be willing to accept such a trade so long as $x$ is no more than 2. Thus, $x$ again lies between 1 and 2 if both of us are to be better off from the trade, only now I am giving you bananas in exchange for apples rather than the other way around.

(d) *What would have to be true about our MRS's at our current bundles in order for you not to be able to come up with a mutually beneficial trade?*

Answer: In order for us not to be able to trade in a mutually beneficial way, your $MRS$ at your current bundle would have to be identical to my $MRS$ at my current bundle.

(e) True or False*: If we have different tastes, then we will always be able to trade with both of us benefitting.*

Answer: This statement is generally false. What matters is not that we have different tastes (i.e. different maps of indifference curves). What matters instead is that, at our current consumption bundle, we value goods differently — that at our current bundle, our $MRS$'s are different. It is quite possible for us to have different tastes (i.e. different maps of indifference curves) but to also be at bundles where our $MRS$ is the same. In that case, we would have the same tastes *at the margin* even though we have different tastes overall (i.e. different indifference maps.)

(f) True or False*: If we have the same tastes, then we will never be able to trade with both of us benefitting.*

Answer: False. People with the same tastes but different bundles of goods may well have different marginal rates of substitution at their current bundles — and this opens the possibility of trading with benefits for both sides.

**B:** *Consider the following five utility functions and assume that $\alpha$ and $\beta$ are positive real numbers:*

$$\begin{aligned}
&1.\ u^A(x_1,x_2) = x_1^{\alpha}x_2^{\beta}\\
&2.\ u^B(x_1,x_2) = \alpha x_1 + \beta x_2\\
&3.\ u^C(x_1,x_2) = \alpha x_1 + \beta \ln x_2\\
&4.\ u^D(x_1,x_2) = \left(\frac{\alpha}{\beta}\right)\ln\, x_1 + \ln x_2\\
&5.\ u^E(x_1,x_2) = -\alpha \ln x_1 - \beta \ln x_2
\end{aligned} \tag{4.11}$$

(a) *Calculate the formula for MRS for each of these utility functions.*

Answer: These would be

$$\begin{aligned}
&1.\ MRS^A = -\frac{\alpha x_1^{\alpha-1}x_2^{\beta}}{\beta x_1^{\alpha}x_2^{\beta-1}} = -\frac{\alpha x_2}{\beta x_1}\\
&2.\ MRS^B = -\frac{\alpha}{\beta}\\
&3.\ MRS^C = -\frac{\alpha}{\beta/x_2} = -\frac{\alpha x_2}{\beta}\\
&4.\ MRS^D = -\frac{\alpha/(\beta x_1)}{1/x_2} = -\frac{\alpha x_2}{\beta x_1}\\
&5.\ MRS^E = -\frac{-\alpha/x_1}{-\beta/x_2} = -\frac{\alpha x_2}{\beta x_1}
\end{aligned} \tag{4.12}$$

(b) *Which utility functions represent tastes that have linear indifference curves?*

Answer: Linear indifference curves are indifference curves that have the same slope everywhere — i.e. indifference curves with constant rather than diminishing $MRS$. Thus, the $MRS$ cannot depend on $x_1$ or $x_2$ for the indifference curve to be linear — which is the case only for $u^B(x_1,x_2)$.

(c) *Which of these utility functions represent the same underlying tastes?*

Answer: Two conditions have to be met for utility functions to represent the same tastes: (1) the indifference curves they give rise to must have the same shapes, and (2) the numbering on the indifference curves needs to have the same order (though not the same magnitude.) To check that indifference curves from two utility functions have the same shape, we have to check that the $MRS$ for those utility functions are the same. This is true for $u^A$, $u^D$ and $u^E$. To check that the ordering of the numbers associated with indifference curves goes in the same direction, we need to go back to the utility functions. In $u^A$, for instance, more of $x_1$ and/or $x_2$ means higher utility values. The same is true for $u^D$. Thus $u^A$ and $u^D$ represent the same underlying tastes because they give rise to the same shapes for all the indifference curves and both have increasing numbers associated with indifference curves as we move northeast in the graph of the indifference curves. But $u^E$ is different: While it gives rise to indifference curves with the same shapes as $u^A$ and $u^D$, the utility values associated with the indifference curves become increasingly negative — i.e. they decline — as we increase $x_1$ and/or $x_2$. Thus, higher numerical labels for indifference curves happen to the southwest rather than the northeast — indicating that less is better than more. So the only two utility functions in this problem that represent the same tastes are $u^A$ and $u^D$.

(d) *Which of these utility functions represent tastes that do not satisfy the monotonicity assumption?*

Answer: As just discussed in the answer to B(c), $u^E$ represents tastes for which less is better than more — because the labeling on the indifference curves gets increasingly negative as we move to the northeast (more of everything) and increasingly less negative as we move toward the origin. In all other cases, more $x_1$ and/or more $x_2$ creates greater utility as measured by the utility functions.

(e) *Which of these utility functions represent tastes that do not satisfy the convexity assumption?*

Answer: As we move to the right on an indifference curve, $x_1$ increases and $x_2$ decreases. We can then look at the formulae for $MRS$ that we derived for each utility function to see what happens to the $MRS$ as $x_1$ increases while $x_2$ decreases. In $MRS^A$, for instance, this would result in a decrease in the numerator and an increase in the denominator — i.e. we are dividing a smaller number by a larger number as $x_1$ increases and $x_2$ decreases. Thus, in absolute value, the $MRS$ declines as we move to the right in our graph — which implies we have diminishing $MRS$ and the usual shape for the indifference curves. Since they share the same $MRS$, the same holds for $u^D$ and $u^E$. For $u^C$, it is similarly true that an increase in $x_1$ accompanied by a decrease in $x_2$ (i.e. a movement along the indifference curve toward the right in the graph) causes the $MRS$ to fall — only this time $x_1$ plays no role and the drop is entirely due to the reduction in the numerator. For $u^B$, the $MRS$ is constant — implying no change in the $MRS$ as we move along an indifference curve to the right in the graph.

We can then conclude the following: $u^B$ satisfies the convexity assumption but barely so — averages are the same as extremes (but not better). Furthermore, $u^A$, $u^C$ and $u^D$ all represent monotonic tastes with diminishing marginal rates of substitution along indifference curves. Thus, averages between extremes that lie on the same indifference curve will be preferred to the extremes because the averages lie to the northeast of some bundles on the indifference curves on which the extremes lie, and, since more is better, this implies the averages are better than the extremes. So $u^A$, $u^C$ and $u^D$ all satisfy the convexity assumption. That leaves only $u^E$ which we concluded before does not satisfy the monotonicity assumption but its indifference curves look exactly like they do for $u^A$ and $u^D$. If you pick any two bundles on an indifference curve, it will therefore again be true that the average of those bundles lies to the northeast of some of the bundles on that indifference curve — but now a movement to the northeast makes the individual worse off, not better off. Thus, averages are worse than extremes for the tastes represented by $u^E$ — which implies that $u^E$ represents tastes that are neither convex nor monotonic.

(f) *Which of these utility functions represent tastes that are not rational (i.e. that do not satisfy the completeness and transitivity assumptions)?*

Answer: Each of these is a function that satisfies the mathematical properties of functions. In each case, you can plug in any bundle $(x_1, x_2)$ and the function will assign a utility value. Thus, any two bundles can be compared — and completeness is satisfied. Furthermore, it is mathematically not possible for a function to assign a value to bundle $A$ that is higher than the value it assigns to a different bundle $B$ which in turn is higher than the value assigned to a third bundle $C$ — without it also being true that the value assigned to $C$ is lower than the value assigned to $A$. Thus, transitivity is satisfied.

(g) *Which of these utility functions represent tastes that are not continuous?*

Answer: All the functions are continuous without sudden jumps — and therefore represent tastes that are similarly continuous.

(h) *Consider the following statement: "Benefits from trade emerge because we have different tastes. If individuals had the same tastes, they would not be able to benefit from trading with one another." Is this statement ever true, and if so, are there any tastes represented by the utility functions in this problem for which the statement is true?*

Answer: What we found in our answers in part A is that, in order for individuals to be able to benefit from trading, it must be the case that their indifference curves through their current consumption bundle have different slopes. It does not matter whether their indifference maps are identical. So long as they are at different current bundles that have different $MRS$'s, mutually beneficial trades are possible. You and I, for instance, might have identical tastes over apples and bananas, but I might have mostly bananas and you might have mostly apples. Then you would probably be willing to trade lots of apples for more bananas, and I'd be willing to let go of bananas pretty easily to get more apples. The only way we cannot benefit from trading with one another is if our $MRS$'s through our current bundle are the same. This might be true for some bundles when we have identical tastes (such as when we currently own the same bundle), but it is not generally true just because we have the same tastes. The only utility function from this problem for which the statement generally holds is therefore $u^B$, the utility function that represents tastes with the same $MRS$ at all bundles. If you and I shared those tastes, then we would have the same $MRS$ regardless of which bundles we currently owned — and this makes it impossible for us to become better off through trade.

The statement in this problem could be re-phrased in a way that would make it universally true for all tastes: "Benefits from trade emerge because we have different tastes *at the margin*" — that is, when we have the same willingness to trade goods off for one another around the bundle we currently consume, then we have the same $MRS$ and can't trade.

## Exercise 4.10: *Investor Tastes over Risk and Return*

Business Application: *Investor Tastes over Risk and Return*: *Suppose you are considering where to invest money for the future.*

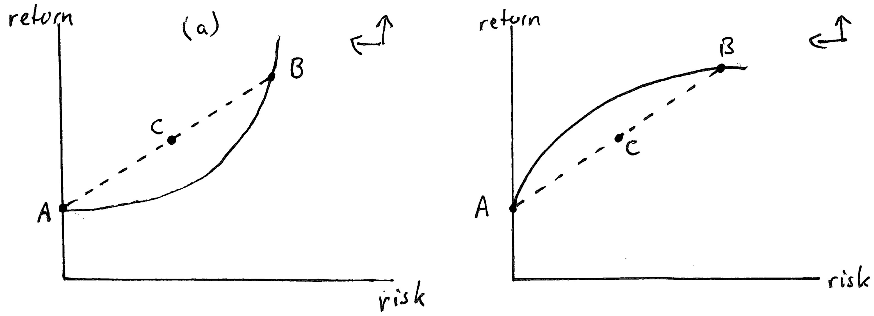
**A:** *Like most investors, you care about the expected return on your investment as well as the risk associated with the investment. But different investors are willing to make different kinds of tradeoffs relative to risk and return.*

(a) *On a graph, put risk on the horizontal axis and expected return on the vertical. (For purposes of this exercise, don't worry about the precise units in which these are expressed.) Where in your graph would you locate "safe" investments like inflation indexed government bonds — investments for which you can predict the rate of return with certainty?*

Answer: Such investments would appear on the vertical axis since risk is represented on the horizontal axis. All such investments have zero risk.

(b) *Pick one of these "safe" investment bundles of risk and return and label it $A$. Then pick a riskier investment bundle $B$ that an investor could plausibly find equally attractive (given that risk is bad in the eyes of investors while expected returns are good).*

Answer: Panel (a) of Graph 4.6 (on the next page) depicts a safe investment $A$ on the vertical axis. An investment bundle with risk must then have a higher return since investors like greater returns and less risk. Put differently, investors become better off moving up and to the left (as indicated by the arrows in the top left of panel (a)), which means that bundles indifferent to $A$ must lie to the northeast.




Graph 4.6: Tastes over Risk and Return

(c) *If your tastes are convex and you only have investments A and B to choose from, would you prefer diversifying your investment portfolio by putting half of your investment in A and half in B?*

Answer: Such diversification would result in bundle $C$ in panel (a) of the graph. Convexity of tastes implies the illustrated shape of the indifference curve through $A$ and $B$ — such that the set of bundles better than $A$ (and $B$) is a convex set. This causes $C$ to lie to the northwest of some of the bundles that are indifferent to $A$ and $B$, which means $C$ must be preferred to $A$ and $B$. So, yes, you would choose to diversify.

(d) *If your tastes are non-convex, would you find such diversification attractive?*

Answer: If tastes are not convex, then $C$ lies below the indifference curve through $A$ and $B$ as illustrated in panel (b) of the graph. Thus, you would not choose to diversify.

**B:** *Suppose an investor has utility function $u(x_1, x_2) = (R - x_1)x_2$ where $x_1$ represents the risk associated with an investment, $x_2$ is the expected return and R is a constant.*

(a) *What is the MRS of risk for return for this investor.*

Answer: The $MRS$ is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{-x_2}{R - x_1} = \frac{x_2}{R - x_1}, \tag{4.13}$$

which is positive for sufficiently high $R$.

(b) *Suppose A is a risk free investment, with $x_1^A = 0$, and suppose that B is risky but our investor is indifferent between A and B. What must the return $x_2^A$ on the risk-free investment be in terms of $x_1^B$ and $x_2^B$?*

Answer: The utility of investment $B$ is $(R - x_1^B)x_2^B$. The utility of investment $A$ is $Rx_2^A$. For the investor to be indifferent, these utilities have to be equal to one another; i.e. $(R - x_1^B)x_2^B = Rx_2^A$. Solving this for $x_2^A$, we get

$$x_2^A = \frac{(R - x_1^B)x_2^B}{R}. \tag{4.14}$$

(c) *Do this investor's tastes satisfy convexity? Illustrate by considering whether this investor would be willing to switch from A or B in part (b) to putting half his investment in A and half in B.*

Answer: The answer is yes, the tastes satisfy convexity — and yes, the person would be willing to diversify by mixing $A$ and $B$. To show this, let's calculate the utility of investment $C$ that mixes $A$ and $B$. The risk characteristic of this investment would be

$$x_1^C = \frac{1}{2}x_1^A + \frac{1}{2}x_1^B = \frac{x_1^B}{2} \tag{4.15}$$

since $x_1^A = 0$. The expected return of investment $C$ would be

$$x_2^C = \frac{1}{2}x_2^A + \frac{1}{2}x_2^B = \frac{1}{2}\left(\frac{(R-x_1^B)x_2^B}{R}\right) + \frac{1}{2}x_2^B = \frac{2Rx_2^B - x_1^B x_2^B}{2R}. \tag{4.16}$$

The utility from $C$ is then

$$u(x_1^C, x_2^C) = (R - x_1^C)x_2^C = \left(R - \frac{x_1^B}{2}\right)\left(\frac{2Rx_2^B - x_1^B x_2^B}{2R}\right) = \frac{4R^2 x_2^B - 4Rx_1^B x_2^B + (x_1^B)^2 x_2^B}{4R} \tag{4.17}$$

which reduces to

$$\begin{aligned} u(x_1^C, x_2^C) = & \ Rx_2^B - x_1^B x_2^B + \frac{(x_1^B)^2 x_2^B}{4R} = \\ & = \left(R - x_1^B\right)x_2^B + \frac{(x_1^B)^2 x_2^B}{4R} = u(x_1^B, x_2^B) + \frac{(x_1^B)^2 x_2^B}{4R}. \end{aligned} \tag{4.18}$$

The utility of $C$ is therefore equal to the utility of $B$ plus a positive amount; i.e. the utility of $C$ is strictly greater than the utility of $B$. Put differently, the average of $A$ and $B$ is strictly preferred.

(d) *Suppose $R = 10$ for our investor. Imagine he is offered the following 3 investment portfolios: (1) a no-risk portfolio of government bonds with expected return of 2 and 0 risk; (2) a high risk portfolio of volatile stocks with expected return of 10 and risk of 8; or a portfolio that consists half of government bonds and half of volatile stocks, with expected return of 6 and risk of 4. Which would he choose?*

Answer: By plugging in these risk and return values into the utility function $u(x_1, x_2) = (10 - x_1)x_2$, we get utility of 20 for the no-risk portfolio, utility of 20 for the high risk portfolio and utility 36 for the mixed portfolio. He would choose the mixed portfolio.

(e) *Suppose a second investor is offered the same three choices. This investor is identical to the first in every way, except that $R$ in his utility function is equal to 20 instead of 10. Which portfolio will he choose?*

Answer: Plugging the risk and return numbers into $u(x_1, x_2) = (20 - x_1)x_2$, we now get utility of 40 for the risk free portfolio, utility of 120 for the high risk portfolio and utility of 96 for the mixed portfolio. He will choose the high risk portfolio.

(f) True or False*: The first investor's tastes are convex while the second one's are not.*

Answer: False. It's easy to see that the first investor's tastes are indeed convex — the no risk and high risk portfolios have the same utility value, and the average between them has a higher utility value. Averages are better than extremes for the first investor. For the second investor, the average is not better than the extremes — but the extremes are not on the same indifference curve. Convexity only says that the average of indifferent bundles is better than the extremes — not the average between any two bundles. And we showed in part B(c), that tastes represented by the utility function $u(x_1, x_2) = (R - x_1)x_2$ are convex regardless of the value of $R$ — so the second investor's tastes are also convex.

(g) *What value of $R$ would make the investor choose the no-risk portfolio?*

Answer There are many values of $R$ that would do this. For instance, if $R = 5$, the utility of the risk free portfolio is 10 while the utility of the risky portfolio is −30 and the utility of the mixed portfolio is 6. For $R = 6$, the utility of the no risk portfolio is 12, as is the utility of the mixed portfolio (with the risky portfolio getting utility −20). Thus, $R = 6$ is the highest value of $R$ that can justify a choice of the risk-free portfolio.

## Exercise 4.13

*In this exercise, we will explore some logical relationships between families of tastes that satisfy different assumptions.*

**A:** *Suppose we define a strong and a weak version of convexity as follows: Tastes are said to be* strongly convex *if, whenever a person with those tastes is indifferent between $A$ and $B$, she strictly prefers the average of $A$ and $B$ (to $A$ and $B$). Tastes are said to be* weakly convex *if, whenever a person with those tastes is indifferent between $A$ and $B$, the average of $A$ and $B$ is at least as good as $A$ and $B$ for that person.*

(a) *Let the set of all tastes that satisfy strong convexity be denoted as $SC$ and the set of all tastes that satisfy weak convexity as $WC$. Which set is contained in the other? (We would, for instance, say that "$WC$ is contained in $SC$" if any taste that satisfies weak convexity also automatically satisfies strong convexity.)*

Answer: Suppose your tastes satisfy the strong convexity condition. Then you always strictly prefer averages to extremes (where the extremes are such that you are indifferent between them). That automatically means that the average between such extremes is *at least as good as* the extremes — which means that your tastes automatically satisfy weak convexity. Thus, the set $SC$ must be fully contained within the set $WC$.

(b) *Consider the set of tastes that are contained in one and only one of the two sets defined above. What must be true about some indifference curves on any indifference map from this newly defined set of tastes?*

Answer: We already concluded above that all strongly convex tastes are also weakly convex. So tastes that are strongly convex cannot be in the newly defined set because they appear in both $SC$ and $WC$ — and we are defining our new set to contain tastes that are only in one of these sets. The newly defined set therefore contains only tastes that satisfy weak convexity but not strong convexity. The only difference between weak and strong convexity is that the former permits averages to be just as good as extremes while the latter insists that averages are strictly better than extremes. When an average is just as good as two extremes from the same indifference curve, it must be that the line connecting the extremes is all part of the same indifference curve. Thus, some indifference curves in a weakly convex indifference map that lies outside $SC$ must have "flat spots" that are line segments.

(c) *Suppose you are told the following about 3 people: Person 1 strictly prefers bundle $A$ to bundle $B$ whenever $A$ contains more of each and every good than bundle $B$. If only some goods are represented in greater quantity in $A$ than in $B$ while the remaining goods are represented in equal quantity, then $A$ is at least as good as $B$ for this person. Such tastes are often said to be* weakly monotonic. *Person 2 likes bundle $A$ strictly better than $B$ whenever at least some goods are represented in greater quantity in $A$ than in $B$ while others may be represented in equal quantity. Such tastes are said to be* strongly monotonic. *Finally, person 3's tastes are such that, for every bundle $A$, there always exists a bundle $B$ very close to $A$ that is strictly better than $A$. Such tastes are said to satisfy* local nonsatiation. *Call the set of tastes that satisfy strict monotonicity $SM$, the set of tastes that satisfy weak monotonicity $WM$, and the set of tastes that satisfy local non-satiation $L$. What is the relationship between these sets? Put differently, is any set contained in any other set?*

Answer: If your tastes satisfy strong monotonicity, it means that $A$ is strictly preferred to $B$ even if $A$ and $B$ are identical in every way except that $A$ has more of one good than $B$. This means that your tastes would automatically satisfy weak monotonicity — because weak monotonicity only requires that $A$ is at least as good under that condition and thus permits indifference between $A$ and $B$ unless all goods are more highly represented in $A$ than in $B$. All strongly monotone tastes are weakly monotone, which means $SM$ is fully contained in $WM$. Local non-satiation only requires that, for every bundle $A$, there exists some bundle $B$ close to $A$ such that $B$ is preferred to $A$. If your tastes satisfy strong monotonicity, then we know such a bundle always exists: Begin at some $A$ and then add a tiny bit of every good to $A$ to form $B$. As long as we add a tiny bit to all goods, strong monotonicity says $B$ is strictly better than $A$. The same works for weakly monotonic tastes. Thus, both $SM$ and $WM$ are fully contained in $L$. But there are also tastes in $L$ such that these tastes are not in $WM$. Consider tastes where at some bundle $A$ there are no bundles with more goods close to $A$

that are preferred to $A$ but there is a bundle with slightly fewer goods that is preferred to $B$. Then such tastes would satisfy local non-satiation but not weak (or strong) convexity.

(d) *Give an example of tastes that fall in one and only one of these three sets?*

Answer: Since we have just concluded that $SM$ is contained in $WM$ which is contained in $L$, such tastes must satisfy local non-satiation but not weak monotonicity. Consider tastes over labor and consumption. We would generally like to expend less labor and have more consumption. Such tastes are not strongly or weakly monotonic because $A$ is strictly less preferred to $B$ if $A$ contains the same amount of consumption but more labor. But they do satisfy local non-satiation because for every $A$, we can make the person better off through less labor or more consumption (or both).

(e) *What is true of tastes that are in one and only one of the two sets $SM$ and $WM$?*

Answer: Since $SM$ is contained in $WM$, such tastes must be weakly monotonic. (If they were strongly monotonic, they would be contained in both sets). Consider bundles $A$ and $B$ that are identical in every way except that $A$ has more of one of the goods than $B$. For tastes to be weakly monotonic but not strongly monotonic, it must be that there exists such an $A$ and $B$ and that a person with such tastes is indifferent between $A$ and $B$. (If such a person strictly preferred all such $A$ bundles to all such $B$ bundles, her tastes would be strongly monotonic.) Thus, tastes that fall in $WM$ but not $SM$ must have some indifference curves with either horizontal or vertical "flat spots".

**B:** *Below we will consider the logical implications of convexity for utility functions. For the following definitions,* $0 \leq \alpha \leq 1$. *A function* $f : \mathbb{R}^2_+ \rightarrow \mathbb{R}^1$ *is defined to be* quasiconcave *if and only if the following is true: Whenever* $f(x_1^A, x_2^A) \leq f(x_1^B, x_2^B)$*, then* $f(x_1^A, x_2^A) \leq f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right)$. *The same type of function is defined to be* concave *if and only if* $\alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B) \leq f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right)$.

(a) True or False*: All concave functions are quasiconcave but not all quasiconcave functions are concave.*

Answer: True. Suppose we start with a concave function $f$. Then

$$\alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B) \leq f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right). \tag{4.19}$$

Now suppose that $f(x_1^A, x_2^A) \leq f(x_1^B, x_2^B)$. Then it must be true that

$$f(x_1^A, x_2^A) \leq \alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B). \tag{4.20}$$

But that implies that whenever $f(x_1^A, x_2^A) \leq f(x_1^B, x_2^B)$, then

$$f(x_1^A, x_2^A) \leq f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right) \tag{4.21}$$

— which is the definition of a quasi-concave function. Thus, *concavity of a function implies quasi-concavity.*

But the reverse does not have to hold. Suppose that when $\alpha = 0.5$, $f(x_1^A, x_2^A) = 10$, $f(x_1^B, x_2^B) = 100$ and $f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right) = 20$. The condition for quasi-concavity is satisfied — so suppose $f$ is in fact quasi-concave throughout. Notice, however, that $\alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B) = 0.5(10) + (0.5)100 = 55$. Thus,

$$20 = f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right) < \alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B) = 55, \tag{4.22}$$

which directly violates concavity.

An example of a function that is quasi-concave but not concave is $u(x_1, x_2) = x_1^2 x_2^2$.

(b) *Demonstrate that, if $u$ is a quasiconcave utility function, the tastes represented by $u$ are convex.*

Answer: Tastes are convex if averages of bundles over which we are indifferent are better than those bundles. Suppose tastes are represented by $u$ and $u$ is quasiconcave. Pick $A =$

$(x_1^A, x_2^A)$ and $B = (x_1^B, x_2^B)$ such that $u(x_1^A, x_2^A) = u(x_1^B, x_2^B)$. Let bundle $C$ be some weighted average between $A$ and $B$; i.e.

$$C = (x_1^C, x_2^C) = \left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right). \tag{4.23}$$

Then quasiconcavity of $u$ implies that

$$u(x_1^A, x_2^A) \leq u(x_1^C, x_2^C), \tag{4.24}$$

which tells us that the average bundle $C$ is at least as good as the extreme bundles $A$ and $B$ ( since $u(x_1^A, x_2^A) = u(x_1^B, x_2^B)$) that the individual is indifferent between. Thus, *quasiconcavity of the utility function implies convexity of underlying tastes represented by that utility function.*

(c) *Do your conclusions above imply that, if u is a concave utility function, the tastes represented by u are convex?*

Answer: Since we concluded in (a) that all concave functions are quasiconcave, and since we concluded in (b) that all quasiconcave utility functions represent tastes that satisfy convexity, it must be that all concave utility functions also represent tastes that are convex.

(d) *Demonstrate that, if tastes over two goods are convex, any utility functions that represents those tastes must be quasiconcave.*

Answer: Suppose we consider bundle $A = (x_1^A, x_2^A)$ and $B = u(x_1^B, x_2^B)$ over which an individual with convex tastes is indifferent. Any utility function that represents these tastes must therefore be such that $u(x_1^A, x_2^A) = u(x_1^B, x_2^B)$ which makes the statement

$$u(x_1^A, x_2^A) \leq u(x_1^B, x_2^B) \tag{4.25}$$

also true (since the inequality is weak). Now define a weighted average $C$ of bundles $A$ and $B$; i.e.

$$C = (x_1^C, x_2^C) = \left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right). \tag{4.26}$$

Convexity of tastes implies that $C$ is at least as good as $A$. Thus, any utility function that represents these tastes must be such that

$$u(x_1^A, x_2^A) \leq u(x_1^C, x_2^C). \tag{4.27}$$

We have therefore concluded that the utility function representing convex tastes must be such that, whenever $u(x_1^A, x_2^A) \leq u(x_1^B, x_2^B)$, then

$$u(x_1^A, x_2^A) \leq \left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right), \tag{4.28}$$

which is the definition of a quasiconcave function. Thus, *convexity of tastes implies quasiconcavity of any utility function that represents those tastes.*[1]

(e) *Do your conclusions above imply that, if tastes over two goods are convex, any utility function that represents those tastes must be concave?*

Answer: No. We have concluded that convexity of tastes implies quasiconcavity of utility functions and we have shown in (a) that there are quasiconcave utility functions that are *not* concave. So the fact that convexity is represented by quasiconcave utility functions does not imply that convexity requires concave utility functions. In fact it does not — it only requires quasiconcavity.

(f) *Do the previous conclusions imply that utility functions which are not quasiconcave represent tastes that are not convex?*

Answer: Yes. In (d) we showed that convexity *necessarily* means that utility functions will be quasiconcave. Thus, when utility functions are *not* quasiconcave, they cannot represent convex tastes. They must therefore represent non-convex tastes.

[1] We actually showed that this statement holds when $u^A = u^B$ — but the same reasoning holds when $u^A < u^B$.

## Conclusion: Potentially Helpful Reminders

1. Convexity in tastes is easy to recognize when "more is better" — but might be a bit confusing otherwise. Here is a simple trick to check whether the tastes you have drawn are convex: Use two arrows that have the same starting point and indicate which horizontal and vertical direction is "better" for the consumer. (When tastes are monotonic, these point to the right and up.) Convexity then implies that the indifference curves bend toward the corner of the arrows you have drawn. (Graph 4.3 in the answer to within-chapter exercise 4A.10 has an example of this. Another example is in Graph 4.6 in the answer to end-of-chapter exercise 4.11.)

2. It should be reasonably clear that tastes — how we *subjectively feel* about stuff — should not typically depend on prices (which only affect what we can *objectively afford*). Put differently, our *circumstances* are different from our *tastes*. But sometimes that gets a little hazy when circumstances other than the usual budget parameters matter. An example of this is given in end-of-chapter exercise 4.7 where we think of "air safety" as one of the circumstances a consumer cannot himself change.

3. Exercise 4.5 is a good exercise to prepare for some of the ideas that are coming up in Chapter 6 as well as later on in Chapter 16.

4. But the last two end-of-chapter exercises are relatively abstract and probably beyond the level of most (but not all) courses that use this text.