CHAPTER

5

Different Types of Tastes

While Chapter 4 introduced us to a general way of thinking about tastes, Chapter 5 gets much more specific and introduces particular dimensions along which we might differentiate between tastes. In particular, we differentiate tastes based on

1. The curvature of individual indifference curves — or how quickly the MRS changes *along an indifference curve*;

2. The relationships between indifference curves — or how the MRS changes *across indifference curves within an indifference map*; and

3. Whether or not indifference curves *cross horizontal or vertical axes* or whether they *converge to the axes*.

The first of these in turn determines the degree to which consumers are willing to substitute between goods (and will lead to what we call the "substitution effect" in Chapter 7) while the second of these determines how consumer behavior responds to changes in income (and will lead to what we call the "income effect" in Chapter 7). Finally, the third category of taste differences becomes important in Chapter 6 where we will see how corner versus interior optimal solutions for a consumer emerge.

Chapter Highlights

The main points of the chapter are:

1. The **degree of substitutability** or, in part B language, the **elasticity of substitution** for a consumer at a particular consumption bundle arises from the **curvature** of the indifference curve at that bundle. There may be no substitutability (as in perfect complements) or perfect substitutability (perfect substitutes) or an infinite number of cases in between these extremes.

2. **Quasilinearity** and **Homotheticity** of tastes represent special cases that describe how indifference curves from the same map relate to one another. These properties have no direct relationship to the concept of substitutability. Tastes are quasilinear in a good x if the MRS only depends on the level of x consumption (and not the level of other goods' consumption. Tastes are homothetic when the MRS depends only on the relative levels of the goods in a bundle.

3. Sometimes it is reasonable to assume that indifference curves only converge to the axes without ever crossing them; other times we assume that they cross the axes. When an indifference curve crosses an axis, it means that we can gain utility beyond what we have by not consuming even if we consume none of one of the goods. When indifference curves only converge to the axes, then some consumption of all goods is necessary in order for a consumer to experience utility above what she would experience by not consuming at all.

4. If you are reading part B of the chapter, you should begin to understand the family of **constant elasticity of substitution utility functions** — with perfect complements, perfect substitutes and Cobb-Douglas tastes as special cases. You should also be able to demonstrate whether a utility function is homothetic or quasilinear. (Most utility functions we use in this text tend to be one or the other.)

Using the LiveGraphs

For an overview of what is contained on the LiveGraphs site for each of the chapters (from Chapter 2 through 29) and how you might utilize this resource, see pages 2-3 of Chapter 1 of this *Study Guide*. To access the LiveGraphs for Chapter 5, click the *Chapter 5* tab on the left side of the LiveGraphs web site.

In addition to the *Animated Graphics*, *Static Graphics* and *Downloads* portions of the LiveGraphs site, this Chapter has several **Exploring Relationships** modules that illustrate the diversity of tastes that can emerge even from what appear to be very restrictive assumptions. The modules were designed for students who are covering part B of the *Microeconomics: An Intuitive Approach with Calculus* text — but I think you might find them engaging even if you are not formally doing the math. The three modules give you a chance to explore:

1. **Cobb-Douglas** tastes (which could be thought of as lying "halfway between" the extremes of perfect substitutes and perfect complements;

2. **Constant Elasticity of Substitution** tastes which allow for all degrees of substitutability that lie between the extremes ; and

3. **Quasilinea**r tastes.

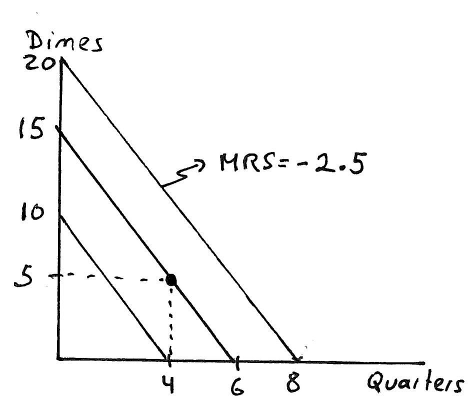
In addition, there are some "games" that require you to determine the relationship between mathematical and graphical concepts. (We are still experimenting with the idea of such games and whether they are useful — and thus looking for feedback on those.)

5A Solutions to Within-Chapter-Exercises for Part A

Exercise 5A.1 *How would the graph of indifference curves change if Coke came in 8 ounce cans and Pepsi came in 4 ounce cans?*

Answer: The indifference curves would then have slope of −2 instead of −1 because you would be willing to trade 2 four ounce cans of Pepsi for 1 eight ounce can of Coke. For instance, the indifference curve that contains 1 can of Coke on the horizontal axis would also contain 2 cans of Pepsi on the vertical as well as half a can of Coke and 1 can of Pepsi. All those combinations contain 16 ounces of soft drink.

Exercise 5A.2 *On a graph with quarters on the horizontal axis and dimes on the vertical, what might your indifference curves look like? Use the same method we just employed to graph my indifference curves for Coke and Pepsi — by beginning with one arbitrary bundle of quarters and dimes (say 4 quarters and 5 dimes) and then asking which other bundles might be just as good.*

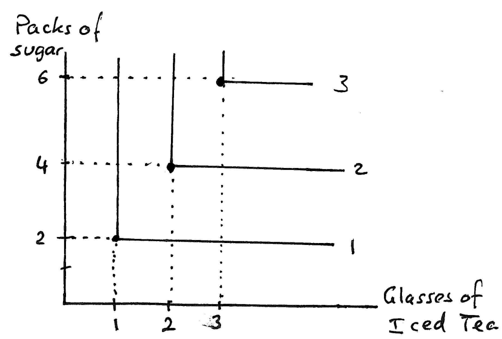


Graph 5.1: Tastes over Dimes and Quarters

Answer: Dimes are worth 10 cents while quarters are worth 25 cents. Thus, you are willing to trade 2.5 dimes for 1 quarter. At 4 quarters and 5 dimes, you have \$1.50. Any other combination of dimes and quarters should be equally desirable. For instance, 15 dimes also make \$1.50, as do 6 quarters. Thus, the indifference curve through the bundle (4,5) has intercept 6 on the horizontal (quarters) axis and 15 on the vertical (dimes) axis. This gives it a slope of -2.5 which is in fact the rate at which we are willing to trade dimes for quarters. This (and two other) indifference curves are depicted in Graph 5.1.

Exercise 5A.3 *What would my wife's indifference curves for packs of sugar and glasses of iced tea look like if she required 2 instead of one packs of sugar for each glass of iced tea?*

Answer: The corners of the indifference curves would now occur at bundles with twice as much sugar as iced tea. For instance, 1 glass of iced tea and 2 packets of sugar make a complete beverage, and no additional sugar and no additional iced tea will make her better off unless she gets more of both. This gives us the indifference curve labeled "1" in Graph 5.2. Similarly, 2 glasses of iced tea and 4 packets of sugar make 2 complete beverages, giving the corner of the indifference curve labeled "2".



Graph 5.2: 2 sugars for each iced tea

Exercise 5A.4 *Suppose I told you that each of the indifference maps graphed in Graph 5.3 corresponded to my tastes for one of the following sets of goods, which pair would you think corresponds to which map? Pair 1: Levi Jeans and Wrangler Jeans; Pair 2: Pants and Shirts; Pair 3: Jeans and Dockers pants.*

Answer: To answer this, we should ask which of the pairs represents goods that seem most substitutable for one another. I would think that would be Pair 1 since that includes two different types of jeans (which many of us probably can't even tell apart easily). Thus, I would think that panel (a) represents Pair 1. We could then ask which of the three pairs represent goods that are most complementary (or least substitutable). Of the remaining pairs, pants and shirts seems less substitutable than Jeans and Dockers pants. Thus Pair 2 — pants and shirts — would correspond to panel (c) where there is the least substitutability between the goods. This leaves panel (b) for Pair 3 — Jeans and Dockers pants.

Exercise 5A.5 *Are my tastes over Coke and Pepsi as described in Section 5A.1 homothetic? Are my wife's tastes over iced tea and sugar homothetic? Why or why not?*

Answer: Yes, both are homothetic. Homothetic tastes are tastes such that the *MRS* is the same along any ray from the origin. For perfect substitutes like Coke and Pepsi, the *MRS* is the same everywhere — which means it is certainly the same along any ray from the origin. For perfect complements like sugar and iced tea, it is easy to also see that the slope of the indifference curves does not change along any ray from the origin. Below the 45 degree line (when one pack of sugar goes with one iced tea), the indifference curve is flat along any ray from the origin; above the 45 degree line, the indifference curve is vertical along any ray from the origin. On the 45 degree line, there is no slope since this is where all the corners of the indifference curve lie. (Since the slope is technically undefined for parts of the indifference map for perfect complements, you can think of this instead as the limit of a sequence of indifference maps that graphs increasingly complementary goods — with each of the maps in the sequence having the characteristic that the *MRS* is unchanged along any ray from the origin.)

Exercise 5A.6 *Are my tastes over Coke and Pepsi as described in Section 5A.1 quasilinear? Are my wife's tastes over iced tea and sugar quasilinear? Why or why not?*

Answer: Tastes are quasilinear in the good on the horizontal axis if the *MRS* is unchanged along any vertical line emanating from the horizontal axis. (Alternatively, tastes are quasilinear in the good on the vertical axis if the *MRS* is unchanged along any horizontal line emanating from the vertical axis.) For perfect substitutes like Coke and Pepsi, the *MRS* is the same everywhere — which means it is certainly the same along any vertical or horizontal line. Thus, perfect substitutes are quasilinear in both goods. Perfect complements like tea and sugar, on the other hand, are not quasilinear in either good. Along any vertical line emanating from the horizontal axis, the indifference curve at some point changes from being horizontal to vertical. (The reverse is true for any horizontal line emanating from the vertical axis). You can also again think of the indifference maps that come closer and closer to those of perfect complements and treat perfect complements as the limiting case. For all maps that approach those of perfect complements, the slopes of indifference curves change along vertical and horizontal lines. Thus neither of the goods is quasilinear.

Exercise 5A.7 *Can you explain why tastes for perfect substitutes are the only tastes that are both quasilinear and homothetic?*

Answer: Quasilinearity implies that the MRS does not change along any vertical line emanating from the horizontal axis (or along any horizontal line emanating from the vertical axis). Homotheticity implies that the MRS is constant along any ray from the origin. Consider any vertical line emanating from the horizontal axis. All rays emanating from the origin pass through that line at some point. So if the MRS has to be the same along the vertical line and it has to he the same along rays from the origin, it must be that the MRS is the same everywhere. (The same is true if we instead considered a horizontal line emanating from the vertical axis when the good on the vertical axis is quasilinear). And the only tastes for which the MRS is the same everywhere are those of perfect substitutes.

Exercise 5A.8 True or False*: Quasilinear goods are never essential.*

Answer: True — or at least almost true. Indifference curves for quasilinear goods (almost always) cross the vertical and horizontal axes. You can think of it this way: if either good was essential — i.e. if the indifference curves did not cross one of the axes, then it must be that they are converging to that axis. If that axis is the vertical axis, then we can draw a vertical line close to the axis and, since all indifference curves are converging, it can't be that the MRS's are the same along the vertical line. Similarly, if that axis is the horizontal axis, then converging indifference curves cannot (at least at some point) have the property that they have the same MRS on a vertical line in the graph. The only reason for the "almost" caveat is the following: Suppose the good on the horizontal axis is a "neutral" good in the sense that you do not care one way or another how much of the good you have — more simply keeps you just as well off as less. The indifference curves would then be horizontal lines — lines that cross the vertical axis but not the horizontal axis. This would make the good on the vertical axis essential (while making the good on the horizontal axis not essential). And the tastes would be quasilinear in the sense that the MRS is the same everywhere. But this is an extreme case — if you add even a slight slope to the indifference curves, they will again cross the horizontal axis (assuming that good is quasilinear) — which would again make the good on the vertical axis non-essential.

5B Solutions to Within-Chapter-Exercises for Part B

Exercise 5B.1 *Calculate the same approximate elasticity of substitution for the indifference curve in Graph 5.7b.*

Answer: The ratio (x_2/x_1) changes from $10/2 = 5$ to $8/4 = 2$. The percentage change in this ratio is therefore $-3/5 = -0.6$. The percentage change in the MRS is again 0.5. Thus, the elasticity of substitution is $(0.6/0.5) = 1.2$.

Exercise 5B.2 *What numerical labels would be attached to the 3 indifference curves in Graph 5.1 by the utility function in equation (5.2)?*

Answer: Each indifference curve would have the label equal to its vertical (or horizontal) intercept; i.e. 1 for the lowest, 2 for the middle and 3 for the highest indifference curve in the graph.

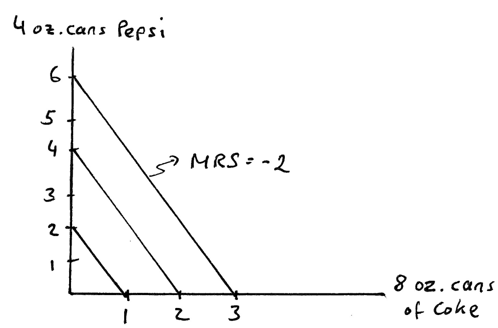
Exercise 5B.3 *Suppose you measured coke in 8 ounce cans and Pepsi in 4 ounce cans. Draw indifference curves and find the simplest possible utility function that would give rise to those indifference curves.*

Answer: Such indifference curves are drawn in Graph 5.3 where the consumer is willing to trade 2 (4 oz) cans of Pepsi for 1 (8 oz) can of Coke — leading to slopes of −2 when Coke is graphed on the horizontal axis. You therefore get twice as much happiness from a can of Coke as from a can of Pepsi, which implies one way of representing these tastes is

$$u(x_1, x_2) = 2x_1 + x_2. \tag{5.1}$$

You can check that the MRS in this case is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{2}{1} = -2. \tag{5.2}$$



Graph 5.3: 8 oz Coke and 4 oz Pepsi

Exercise 5B.4 *Can you use similar reasoning to determine the elasticity of substitution for the utility function you derived in exercise 5B.3?*

Answer: The exact same reasoning holds for all indifference maps with linear indifference curves. Again, it is easiest to think of an indifference map that is close to linear everywhere — and then to think what happens as such an indifference map approaches that of perfectly linear indifference curves. For indifference curves that are close to those with $MRS = -2$ everywhere, we can start at a bundle A with little x_1 and a lot x_2. Even a small change in the MRS will result in a large move down that indifference curve. Thus, the percentage change in the ratio of the goods (which is the numerator in the elasticity of substitution equation) is large for a small percentage change in the MRS (which is the denominator in the elasticity equation). In the limit, I can get larger and larger changes in this numerator with smaller and smaller changes in the denominator as the indifference curve gets closer and closer to being linear. Thus, in the limit the elasticity of substitution is ∞.

Exercise 5B.5 *Plug the bundles* $(3,1)$, $(2,1)$, $(1,1)$, $(1,2)$ *and* $(1,3)$ *into this utility function and verify that each is shown to give the same "utility" — thus lying on the same indifference curve as plotted in Graph 5.2. What numerical labels does this indifference curve attach to each of the 3 indifference curves in Graph 5.2?*

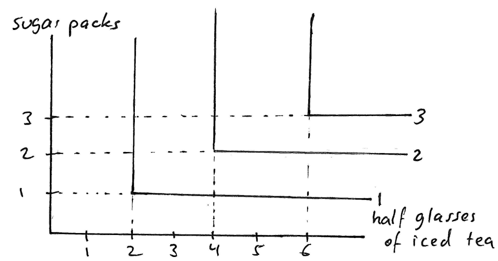
Answer: In each of these bundles, the utility function $u(x_1,x_2) = \min\{x_1,x_2\}$ picks the lower of the two quantities and assigns that number as the utility level of that bundle. Since the lower of the two values in each of these bundles is 1, each is assigned a utility value of 1. The values assigned to the three indifference curves are 1, 2 and 3 — the values of the vertical and horizontal coordinates at the corners of each indifference curve.

Exercise 5B.6 *How would your graph and the corresponding utility function change if we measured iced tea in "half glasses" instead of glasses.*

Answer: In that case, the perfect beverage requires 1 pack of sugar for every 2 units (half glasses) of iced tea. Any more sugar for 2 units of iced tea would add no further utility unless more tea was added as well, and more tea for 1 pack of sugar would not add more utility unless more sugar was added as well. Thus, the indifference curves representing the same tastes as before would look as in Graph 5.4, with the corner points now lying on a ray from the origin that lies below the 45 degree line. A utility function that results in the labeling of the indifference curves that arises in this graph is $u(x_1,x_2) = \min\{0.5x_1, x_2\}$.

Exercise 5B.7 *Can you determine intuitively what the elasticity of substitution is for the utility function you defined in exercise 5B.6?*

Answer: It is again easiest to do this for tastes that are very close to those we graphed in Graph 5.4 but without the sharp kink. Pick A a bit above the ray on which the corners of the indifference curves lie — with the ratio of x_1/x_2 just above 0.5. Then imagine moving to a shallower slope of the indifference curve that contains A. Because of the large curvature of the indifference curve around the the ray that connects the corners of the indifference curves, even a relatively large change in the MRS will not cause us to have to slide very far along the indifference curve — implying a relatively modest change in the ratio x_1/x_2. Thus, for a large percentage change in the MRS (which is the denominator in the elasticity equation), we



Graph 5.4: Half Glasses of tea and full packs of sugar

get a relatively small change in the ratio x_1/x_2 (which is the denominator in the elasticity equation.) As the indifference curve gets closer and closer to that of perfect complements, the percentage change in the consumption good ratio will fall for any percentage change in the MRS — and will approach 0 as the indifference curve approaches that of perfect complements. Thus, the numerator in the elasticity equation approaches zero — leaving us with an elasticity of substitution of zero in the limit.

Exercise 5B.8 *Demonstrate that the functions u and v both give rise to indifference curves that exhibit the same shape by showing that the MRS for each function is the same.*

Answer: The MRS of $v = x_1^{\alpha} x_2^{1-\alpha}$ is

$$MRS^v = -\frac{\partial v/\partial x_1}{\partial v/\partial x_2} = -\frac{\alpha x_1^{\alpha-1} x_2^{1-\alpha}}{(1-\alpha) x_1^{\alpha} x_2^{-\alpha}} = -\frac{\alpha x_2}{(1-\alpha) x_1}, \tag{5.3}$$

and, since $\alpha = \gamma/(\gamma+\delta)$, this can also be written as

$$MRS^v = -\frac{\alpha x_2}{(1-\alpha) x_1} = -\frac{(\gamma/(\gamma+\delta)) x_2}{(1-\gamma/(\gamma+\delta)) x_1} = -\frac{(\gamma/(\gamma+\delta)) x_2}{(\delta/(\gamma+\delta)) x_1} = -\frac{\gamma x_2}{\delta x_1}. \tag{5.4}$$

The MRS of the function $u(x_1, x_2) = x_1^{\gamma} x_2^{\delta}$ is

$$MRS^u = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{\gamma x_1^{\gamma-1} x_2^{\delta}}{\delta x_1^{\gamma} x_2^{\delta-1}} = -\frac{\gamma x_2}{\delta x_1}. \tag{5.5}$$

Thus, $MRS^v = MRS^u$, which implies the indifference curves arising from the two utility functions are identical.

Exercise 5B.9 *Derive the MRS for the Cobb-Douglas utility function and use it to show what happens to the slope of indifference curves along the 45-degree line as α changes.*

Answer: The MRS for the Cobb-Douglas function which is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{\alpha x_1^{(\alpha-1)} x_2^{(1-\alpha)}}{(1-\alpha)x_1^{\alpha} x_2^{-\alpha}} = -\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{x_2}{x_1}\right). \quad (5.6)$$

Along the 45-degree line, $x_1 = x_2$ — which implies $x_2/x_1 = 1$ and the MRS along the 45 degree line is simply $-\alpha/(1-\alpha)$. Thus, when $\alpha = 0.5$, the MRS along the 45 degree line is exactly -1. When $\alpha > 0.5$, the MRS on the 45 degree line is greater than 1 in absolute value, and when $\alpha < 0.5$, the MRS is less than 1 in absolute value along the 45 degree line.

Exercise 5B.10 *What is the elasticity of substitution in each panel of Graph 5.10?*

Answer: The elasticity of substitution for CES utility functions is $\sigma = 1/(1+\rho)$. Thus, the $\rho = -0.8$ in panel (a) translates to $\sigma = 5$; the $\rho = -0.2$ in panel (b) translates to $\sigma = 1.25$; and the $\rho = 2$ in panel (c) translates to $\sigma = 0.33$.

Exercise 5B.11 *Can you describe what happens to the slopes of the indifference curves on the 45 degree line, above the 45 degree line and below the 45 degee line as ρ becomes large (and as the elasticity of substitution therefore becomes small)?*

Answer: The slopes of the indifference curves are described by the MRS which is given by

$$MRS = -\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{x_2}{x_1}\right)^{\rho+1}. \quad (5.7)$$

First, consider bundles on the 45-degree line where $x_1 = x_2$ and thus $x_2/x_1 = 1$. In this case, the second term in the equation remains 1 as ρ gets large — and the MRS therefore stays constant at $-\alpha/(1-\alpha)$.

Next, consider a bundle above the 45 degree line — i.e. a bundle such that $x_1 < x_2$. This implies that $x_2/x_1 > 1$ — which means the second term in the MRS equation increases as ρ gets large. Thus, as ρ gets large, the slope of indifference curves above the 45-degree line become steeper (approaching vertical lines as ρ approaches infinity.)

Finally, suppose we consider a bundle below the 45 degree line — i.e. a bundle such that $x_1 > x_2$. This implies $x_2/x_1 < 1$ — which implies that the second term in the MRS equation decreases as ρ gets large. Thus, the slopes of indifference curves get shallower below the 45 degree line (approaching horizontal lines as ρ approaches infinity).

Thus, as ρ approaches infinity (and as the elasticity of substitution therefore approaches 0), the slopes of indifference curves along the 45 degree line remain unchanged while they flatten out below the 45 degree line and straighten up above the 45 degree line. In other words, as ρ gets large, the shape of the indifference curves approach those of perfect complements.

Exercise 5B.12 *On the "Exploring Relationships" animation associated with Graph 5.10, develop an intuition for the role of the α parameter in CES utility functions and compare those to what emerges in Graph 5.9.*

Answer: No particular answer here — the animated version should illustrate how changing α alters the shapes of indifference curves in ways that should seem familiar from our Cobb-Douglas example in the text.

Exercise 5B.13 *Show that, when we normalize the exponents of the Cobb-Douglas utility function to sum to 1, the function is homogeneous of degree 1.*

Answer: Using the utility function $u(x_1, x_2) = x_1^{\alpha} x_2^{(1-\alpha)}$,

$$u(tx_1, tx_2) = (tx_1)^{\alpha}(tx_2)^{(1-\alpha)} = t^{\alpha} x_1^{\alpha} t^{(1-\alpha)} x_2^{(1-\alpha)} = t x_1^{\alpha} x_2^{(1-\alpha)} = t u(x_1, x_2). \quad (5.8)$$

Exercise 5B.14 *Consider the following variant of the CES function that will play an important role in producer theory: $f(x_1, x_2) = \left(\alpha x_1^{-\rho} + (1-\alpha) x_2^{-\rho}\right)^{-\beta/\rho}$. Show that this function is homogeneous of degree β.*

Answer:

$$\begin{aligned} f(tx_1, tx_2) &= \left(\alpha(tx_1)^{-\rho} + (1-\alpha)(tx_2)^{-\rho}\right)^{-\beta/\rho} = \\ &= \left(t^{-\rho}(\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho})\right)^{-\beta/\rho} = t^{\beta}\left(\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho}\right)^{-\beta/\rho} = \\ &= t^{\beta} f(x_1, x_2). \end{aligned} \quad (5.9)$$

Exercise 5B.15 *Can you demonstrate, using the definition of a homogeneous function, that it is generally possible to transform a function that is homogeneous of degree k to one that is homogeneous of degree 1 in the way suggested above?*

Answer: Suppose a function $f(x_1, x_2)$ is homogeneous of degree k. Then this implies that $f(tx_1, tx_2) = t^k f(x_1, x_2)$ for any $t > 0$. Now consider the function

$$v(x_1, x_2) = \left(f(x_1, x_2)\right)^{1/k}. \quad (5.10)$$

Then

$$\begin{aligned} v(tx_1, tx_2) &= \left(f(tx_1, tx_2)\right)^{1/k} = \left(t^k f(x_1, x_2)\right)^{1/k} \\ &= t\left(f(x_1, x_2)\right)^{1/k} = t v(x_1, x_2). \end{aligned} \quad (5.11)$$

Thus, $v(tx_1, tx_2) = tv(x_1, x_2)$ which is the definition of a function that is homogeneous of degree 1.

Exercise 5B.16 *Use the mathematical expression for quasilinear tastes to illustrate that neither good is essential if tastes are quasilinear in one of the goods.*

Answer: If tastes are quasilinear in x_1, then we can represent them by a function

$$u(x_1, x_2) = v(x_1) + x_2. \tag{5.12}$$

At the bundle (0,0), this would result in utility of $u(0,0) = v(0)$. If the consumer consumes $(x_1,0)$ — i.e. if she consumes only x_1 but no x_2, he utility is $u(x_1,0) = v(x_1)$ which is greater than $v(0)$ which she gets by consuming nothing. Thus, the consumer can get more utility by consuming only x_1 than she could by consuming nothing — which implies that x_2 is not essential. Similarly, if she consumes a bundle $(0,x_2)$ — i.e. if she consumes only x_2 and no x_1, she gets utility $u(0,x_2) = v(0) + x_2$ which is also greater than $u(0,0) = v(0)$. Thus, x_1 is not essential.

Exercise 5B.17 *Show that both goods are essential if tastes can be represented by Cobb-Douglas utility functions.*

Answer: Suppose tastes can be represented by $u(x_1, x_2) = x_1^{\alpha} x_2^{(1-\alpha)}$. Then the utility from consuming (0,0) is $u(0,0) = 0$. Now consider the utility from a bundle $(x_1,0)$ — i.e. a bundle with no x_2 consumption. Utility from such a bundle is $u(x_1,0) = x_1^{\alpha}(0) = 0$ — exactly what it is when the consumer doesn't consume anything at all. Thus, x_2 is essential. By similar reasoning, x_1 is essential.

Exercise 5B.18 *Can you demonstrate similarly that $\sigma = 1$ for the Cobb-Douglas utility function $u(x_1, x_2) = x_1^{\alpha} x_2^{(1-\alpha)}$?*

Answer: We know from our previous work that the MRS of a Cobb-Douglas utility function of this type is $MRS = -(\alpha x_2)/((1-\alpha)x_1)$. Taking absolute values of both sides and solving for (x_2/x_1), we get

$$\frac{x_2}{x_1} = \frac{(1-\alpha)}{\alpha}|MRS|, \tag{5.13}$$

and taking logs,

$$\ln \frac{x_2}{x_1} = \ln|MRS| + \ln \frac{(1-\alpha)}{\alpha}. \tag{5.14}$$

We can then apply the elasticity formula from the appendix to get

$$\sigma = \frac{d \ln(x_2/x_1)}{d \ln|MRS|} = 1. \tag{5.15}$$

End of Chapter Exercises

Exercise 5.4

Suppose two people want to see if they could benefit from trading with one another in a 2-good world.

A: *In each of the following cases, determine whether trade might benefit the two individuals:*

(a) *As soon as they start talking with one another, they find that they own exactly the same amount of each good as the other does.*

Answer: This should in general not keep them from being able to gain from trading with one another as long as their tastes differ at the margin at the bundle that they own. What matters for gains from trade is whether there are differences in the two individual's MRS at the bundle they currently own.

(b) *They discover that they are long-lost twins who have identical tastes.*

Answer: Again, that should not generally keep them from being able to trade with one another, at least as long as they don't currently own the same bundle. People with the same map of indifference curves will typically have different MRS's when they own different bundles — and it is this difference in tastes at the margin that may arise even if people have the same map of indifference curves.

(c) *The two goods are perfect substitutes for each of them — with the same MRS within and across their indifference maps.*

Answer: In this case, there is no way to gain from trade — because no matter what bundle each of the individuals currently owns, their MRS is the same across the two individuals.

(d) *They have the same tastes, own different bundles of goods but are currently located on the same indifference curve.*

Answer: As long as averages are better than extremes, they will be able to trade toward a more "average" bundle and thus will both benefit from trade.

B: *Suppose that the two individuals have CES utility functions, with individual 1's utility given by* $u(x_1,x_2)=\left(\alpha x_1^{-\rho}+(1-\alpha)x_2^{-\rho}\right)^{-1/\rho}$ *and individual 2's by* $v(x_1,x_2)=\left(\beta x_1^{-\rho}+(1-\beta)x_2^{-\rho}\right)^{-1/\rho}$.

(a) *For what values of α, β and ρ is it the case that owning the same bundle will always imply that there are no gains from trade for the two individuals.*

Answer: Owning the same bundle implies identical MRS's for the two individuals only if tastes are the same. This implies that $\alpha=\beta$ (since both utility functions already share the same ρ.)

(b) *Suppose $\alpha=\beta$ and the two individuals therefore share the same preferences. For what values of $\alpha=\beta$ and ρ is it the case that the two individuals are not able to gain from trade regardless of what current bundles they own?*

Answer: When individuals have identical tastes but different current bundles of goods, the only way we know that they cannot trade for sure is if the two goods are in fact perfect substitutes for them (because then their MRS is in fact the same regardless of what bundles they own). This occurs when $\rho=-1$.

(c) *Suppose that person 1 owns twice as much of all goods as person 2. What has to be true about α, β and ρ for them not to be able to trade?*

Answer: The tastes are homothetic — which means that the MRS is the same along any ray from the origin within a single indifference map. If the two indifference maps are furthermore identical, then the same ray from the origin will be associated with the same MRS across the two individuals. If person 1 owns twice as much of everything as person 2, then their current bundles lie on a single ray from the origin — which implies that if the two indifference maps are identical, the two individuals will not be able to trade. This is true if $\alpha=\beta$ for any ρ between -1 and infinity.

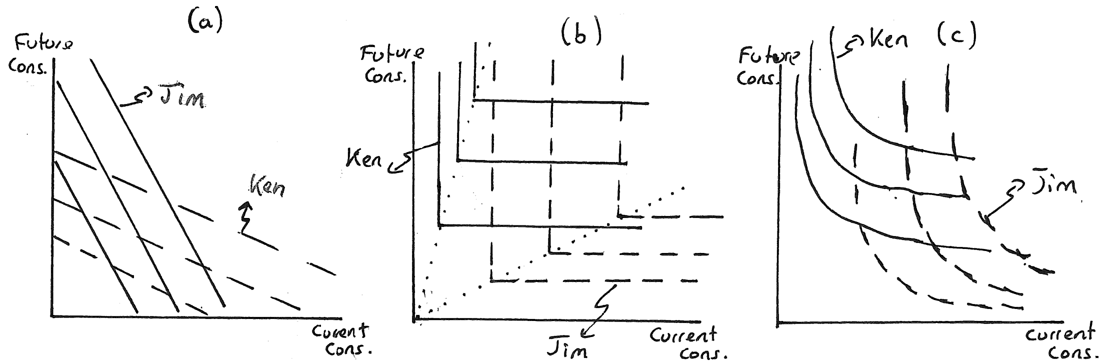
Exercise 5.6

Everyday Application: *Thinking About Old Age*: *Consider two individuals who take a very different view of life — and consider how this shapes their tastes over intertemporal tradeoffs.*

A: *Jim is a 25 year-old athlete who derives most of his pleasure in life from expensive and physically intense activities — mountain climbing in the Himalayas, kayaking in the Amazon, bungee jumping in New Zealand, Lion safaris in Africa and skiing in the Alps. He does not look forward to old age* *when he can no longer do this and plans on getting as much fun in early on as he can. Ken is quite different — he shuns physical activity but enjoys reading in comfortable surroundings. The more he reads, the more he wants to read and the more he wants to retreat to luxurious libraries in the comfort of his home. He looks forward to quiet years of retirement when he can do what he loves most.*

(a) *Suppose both Jim and Ken are willing to perfectly substitute current for future consumption — but at different rates. Given the descriptions of them, draw two different indifference maps and indicate which is more likely to be Jim's and which is more likely to be Ken's.*

Answer: Panel (a) of Graph 5.5 illustrates two indifference maps in one graph — with Jim's indifference curves in solid lines and Ken's in dashed lines. Since Jim is more interested in focusing his consumption now, his *MRS* is larger in absolute value — i.e. he is willing give up more future consumption for current consumption.



Graph 5.5: Jim and Ken's Intertemporal Tastes

(b) *Now suppose neither Jim nor Ken are willing to substitute at all across time periods. How would their indifference maps differ now given the descriptions of them above?*

Answer: Panel (b) illustrates the case where they are not willing to substitute across time — with Jim's indifference curves again dashed and Ken's solid. Even though they are not willing to substitute across time, knowing that Jim wants to consume more now while Ken wants to postpone tells us where the corners of the indifference curves are relative to one another.

(c) *Finally, suppose they both allowed for some substitutability across time periods but not as extreme as what you considered in part (a). Again, draw two indifference maps and indicate which refers to Jim and which to Ken.*

Answer: These are now illustrated in panel (c) — with indifference curve similar to those in (b) except that we add some curvature to allow for some (though not complete) substitutability.

(d) *Which of the indifference maps you have drawn could be homothetic?*

Answer: The indifference maps in (a) are definitely homothetic (since they have the same *MRS* within each map. The others can certainly be homothetic. In panel (b), they are in fact clearly drawn as homothetic since the corners of the indifference curves are drawn along rays from the origin. The same could be true for panel (c).

(e) *Can you say for sure if the indifference maps of Jim and Ken in part (c) satisfy the single crossing property (as defined in end-of-chapter exercise 4.9)?*

Answer: You can't say for sure. The way they are drawn in panel (c), it certainly seems like the single crossing property might hold. If the indifference maps are close to those of perfect complements, the single crossing property will, in fact hold. But you can also imagine starting with the indifference maps in panel (a) and bending them slightly — thus creating two points at which indifference curves from the two maps cross one another.

B: *Continue with the descriptions of Jim and Ken as given in part A and let c_1 represent consumption now and let c_2 represent consumption in retirement.*

(a) *Suppose that Jim's and Ken's tastes can be represented by $u^J(c_1, c_2) = \alpha c_1 + c_2$ and $u^K(c_1, c_2) = \beta c_1 + c_2$ respectively. How does α compare to β — i.e. which is larger?*

Answer: The marginal rates of substitution are

$$MRS^J = -\frac{\partial u^J/\partial c_1}{\partial u^J/\partial c_2} = -\alpha \text{ and } MRS^K = -\frac{\partial u^K/\partial c_1}{\partial u^K/\partial c_2} = -\beta. \tag{5.16}$$

Thus, it must be that $\alpha > \beta$ for us to get indifference maps such as those in panel (a) of Graph 5.5.

(b) *How would you similarly differentiate, using a constant α for Jim and β for Ken, two utility functions that give rise to tastes as described in A(b)?*

Answer: Such tastes can be represented by

$$u^J(c_1, c_2) = \min\{\alpha c_1, c_2\} \text{ and } u^K(c_1, c_2) = \min\{\beta c_1, c_2\}, \tag{5.17}$$

where $\alpha < 1 < \beta$. Consider, $u^J(c_1, c_2) = \min\{\alpha c_1, c_2\}$. Suppose, for instance, that $\alpha = 1/2$. This means that Jim's utility for the bundle $(c_1, c_2) = (2x, x)$ would be equal to $u^J(2x, x) = \min\{(1/2)2x, x\} = \min\{x, x\}$. Thus, $(2x, x)$ for different values of x represent the location of all the corners of the indifference curves for Jim — or, put differently, those corners lie on a ray from the origin with slope $(1/2)$. More generally, they will lie on a ray from the origin with slope α for Jim and with slope β for Ken — and from panel (b) in Graph 5.5, we know that Ken's ray has slope greater than 1 (because his corners lie above the 45-degree line) while Jim's ray has slope less than 1 (because his corners lie below the 45-degree line.)

(c) *Now consider the case described in A(c), with their tastes now described by the Cobb-Douglas utility functions $u^J(c_1, c_2) = c_1^{\alpha} c_2^{(1-\alpha)}$ and $u^J(c_1, c_2) = c_1^{\beta} c_2^{(1-\beta)}$. How would α and β in those functions be related to one another?*

Answer: The MRS for the two functions is

$$MRS^J = \frac{\alpha c_2}{(1-\alpha)c_1} \text{ and } MRS^K = \frac{\beta c_2}{(1-\beta)c_1}. \tag{5.18}$$

Recall that the MRS is equal to -1 along the 45 degree line (where $c_1 = c_2$) when $\alpha = 0.5$. For Ken, the indifference curves are shallower than this on the 45 degree line — implying a smaller MRS in absolute value. This can only happen if $\beta < 0.5$. The reverse is true for Jim — implying $\alpha > 0.5$. Thus, $\alpha > \beta$.

(d) *Are all the tastes described by the above utility functions homothetic? Are any of them quasi-linear?*

Answer: Yes, they are all homothetic. The only one that is quasilinear is the one for perfect substitutes in B(a)

(e) *Can you show that the tastes in B(c) satisfy the single crossing property (as defined in end-of-chapter exercise 4.9)?*

Answer: Pick any arbitrary bundle (c_1, c_2). Given that $\alpha > 0.5 > \beta$ (from (c) above), the absolute value of the marginal rates of substitution at that bundle satisfy

$$\left|MRS^J\right| = \frac{\alpha c_2}{(1-\alpha)c_1} > \frac{\beta c_2}{(1-\beta)c_1} = \left|MRS^K\right|. \tag{5.19}$$

Thus, the slope of Jim's indifference curves at every bundle is steeper than that of Ken's indifference curve at that bundle — which means that indifference curves can only be crossing once.

(f) *Are all the functions in B(a)-(c) members of the family of CES utility functions?*

Answer: Yes. As shown in the text, CES utility functions range from perfect substitutes to perfect complements and include Cobb-Douglas tastes.

Exercise 5.11

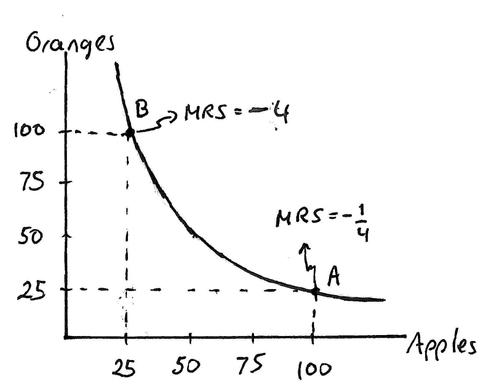
In this exercise, we are working with the concept of an elasticity of substitution. This concept was introduced in part B of the Chapter. Thus, this entire question relates to material from part B, but the A-part of the question can be done simply by knowing the formula for an elasticity of substitution while the B-part of the question requires further material from part B of the Chapter. In Section 5B.1, we defined the elasticity of substitution as

$$\sigma = \left| \frac{\%\Delta(x_2/x_1)}{\%\Delta MRS} \right|. \tag{5.20}$$

A: *Suppose you consume only apples and oranges. Last month, you consumed bundle A=(100,25) — 100 apples and 25 oranges, and you were willing to trade at most 4 apples for every orange. Two months ago, oranges were in season and you consumed B=(25,100) and were willing to trade at most 4 oranges for 1 apple. Suppose your happiness was unchanged over the past two months.*

(a) *On a graph with apples on the horizontal axis and oranges on the vertical, illustrate the indifference curve on which you have been operating these past two months and label the MRS where you know it.*

Answer: This is illustrated in Graph 5.6.



Graph 5.6: Elasticity of Subsitution

(b) *Using the formula for elasticity of substitution, estimate your elasticity of substitution of apples for oranges.*

Answer: This is

$$\sigma = \left| \frac{((100/25)-(25/100))/(100/25)}{(-4-(-1/4))/(-4)} \right| = \left| \frac{(15/4)/4}{(15/4)/4} \right| = 1. \tag{5.21}$$

(c) *Suppose we know that the elasticity of substitution is in fact the same at every bundle for you and is equal to what you calculated in (b). Suppose the bundle C=(50,50) is another bundle that makes you just as happy as bundles A and B. What is the MRS at bundle C?*

Answer: Using B and C in the elasticity of substitution formula, setting σ equal to 1 and letting the MRS at C be denoted by x , we get

$$\left| \frac{((100/25)-(50/50))/(100/25)}{(-4-(-x))/(-4)} \right| = \left| \frac{3/4}{(4-x)/4} \right| = 1, \tag{5.22}$$

and solving this for x, we get $x = 1$ — i.e. the MRS at C is equal to -1.

(d) *Consider a bundle $D = (25,25)$. If your tastes are homothetic, what is the MRS at bundle D?*

Answer: Since it, like bundle C, lies on the 45 degree line, homotheticity implies the MRS is again -1.

(e) *Suppose you are consuming 50 apples, you are willing to trade 4 apples for one orange and you are just as happy as you were when you consumed at bundle D. How many oranges are you consuming (assuming the same elasticity of substitution)?*

Answer: Let the number of oranges be denoted y. Using the bundle $(50, y)$ and $D = (25,25)$ in the elasticity formula and setting it to 1, we get

$$\left|\frac{((50/y)-(25/25))/(50/y)}{(-4-(-1))/(-4)}\right| = \left|\frac{((50/y)-1)/(50/y)}{(3/4)}\right| = 1. \tag{5.23}$$

Solving this for y, we get $y = 12.5$.

(f) *Call the bundle you derived in part (e) E. If the elasticity is as it was before, at what bundle would you be just as happy as at E but would be willing to trade 4 oranges for 1 apple?*

Answer: If the elasticity is 1 from D to E and is again supposed to be 1 from D to this new bundle, there must be symmetry around the 45 degree line (as there was between A and B). At $E = (50, 12.5)$, the MRS is $-1/4$, and the necessary symmetry then means that $MRS = -4$ at (12.5,50).

B: *Suppose your tastes can be summarized by the utility function* $u(x_1, x_2) = \left(\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho}\right)^{-1/\rho}$.

(a) *In order for these tastes to contain an indifference curve such as the one containing bundle A that you graphed in A(a), what must be the value of ρ? What about α?*

Answer: The elasticity of substitution for the CES utility function can be written as $\sigma = 1/(1+\rho)$. Above, we determined that the elasticity of substitution in this problem is 1. Thus, $1 = 1/(1+\rho)$ which implies $\rho = 0$. Since our graph is symmetric around the 45 degree line, it must furthermore be true that $\alpha = 0.5$ — i.e. x_1 and x_2 enter symmetrically into the utility function.

(b) *Suppose you were told that the same tastes can be represented by* $u(x_1, x_2) = x_1^{\gamma} x_2^{\delta}$. *In light of your answer above, is this possible? If so, what has to be true about γ and δ given the symmetry of the indifference curves on the two sides of the 45 degree line?*

Answer: Yes — it is possible because we determined that the elasticity of substitution is 1 everywhere, which is true for Cobb-Douglas utility functions of the form $u(x_1, x_2) = x_1^{\gamma} x_2^{\delta}$. The symmetry implies $\gamma = \delta$.

(c) *What exact value(s) do the exponents γ and δ take if the label on the indifference curve containing bundle A is 50? What if that label is 2,500? What if the label is 6,250,000?*

Answer: If the utility at A is 50, it means $50^{\gamma}50^{\delta} = 50$. Since we just concluded in (a) that $\gamma = \delta$, this implies that $\gamma = \delta = 0.5$. If the utility is 2,500, then $\gamma = \delta = 1$, and if the utility is 6,250,000, $\gamma = \delta = 2$.

(d) *Verify that bundles A, B and C (as defined in part A) indeed lie on the same indifference curve when tastes are represented by the three different utility functions you implicitly derived in B(c). Which of these utility functions is homogeneous of degree 1? Which is homogeneous of degree 2? Is the third utility function also homogeneous?*

Answer: The bundles are A=(100,25), B=(25,100) and C=(50,50). The following equations hold, verifying that these must be on the same indifference curve for each of the three utility functions: $u(x_1, x_2) = x_1^{0.5}x_2^{0.5}$, $v(x_1, x_2) = x_1 x_2$ and $w(x_1, x_2) = x_1^2 x_2^2$:

$$\begin{aligned} u(100,25) &= u(25,100) = u(50,50) = 50 \\ v(100,25) &= v(25,100) = v(50,50) = 2,500 \\ w(100,25) &= w(25,100) = w(50,50) = 6,250,000. \end{aligned} \tag{5.24}$$

The following illustrate the homogeneity properties of the three functions:

$$\begin{aligned} u(tx_1, tx_2) &= (tx_1)^{0.5}(tx_2)^{0.5} = tx_1^{0.5}x_2^{0.5} = tu(x_1, x_2) \\ v(tx_1, tx_2) &= (tx_1)(tx_2) = t^2 x_1 x_2 = t^2 v(x_1, x_2) \\ w(tx_1, tx_2) &= (tx_1)^2(tx_2)^2 = t^4 x_1^2 x_2^2 = t^4 w(x_1, x_2). \end{aligned} \tag{5.25}$$

Thus, u is homogeneous of degree 1, v is homogeneous of degree 2 and w is homogeneous of degree 4.

(e) *What values do each of these utility functions assign to the indifference curve that contains bundle D?*

Answer: Recall that $D = (25,25)$. Thus, the three utility functions assign values of $u(25,25) = 25^{0.5}25^{0.5} = 25$; $v(25,25) = 25(25) = 625$; and $w(25,25) = 25^2(25^2) = 390,625$.

(f) True or False*: Homogeneity of degree 1 implies that a doubling of goods in a consumption basket leads to "twice" the utility as measured by the homogeneous function, whereas homogeneity greater than 1 implies that a doubling of goods in a consumption bundle leads to more than "twice" the utility.*

Answer: This is true. Above, we showed an example of this. More generally, you can see this from the definition of a function that is homogeneous of degree k; i.e. $u(tx_1,tx_2) = t^k u(x_1,x_2)$. Substituting $k = 2$, $u(2x_1,2x_2) = 2^k u(x_1,x_2)$. When $k = 1$ — i.e. when the utility function is homogeneous of degree 1, this implies $u(2x_1,2x_2) = 2u(x_1,x_2)$ — a doubling of goods leads to a doubling of utility assigned to the bundle. More generally, a doubling of goods leads to 2^k times as much utility assigned to the new bundle — and 2^k is greater than 2 when $k > 1$ (and less than 2 when $k < 1$.)

(g) *Demonstrate that the MRS is unchanged regardless of which of the three utility functions derived in B(c) is used.*

Answer: The MRS of a Cobb-Douglas utility function $u(x_1,x_2) = x_1^{\gamma} x_2^{\delta}$ is $MRS = -(\gamma x_2)/(\delta x_1)$ which reduces to $-x_2/x_1$ when $\gamma = \delta$ which is the case for all three of the utility functions above. Thus, the MRS is the same for the three functions.

(h) *Can you think of representing these tastes with a utiliy function that assigns the value of 100 to the indifference curve containing bundle A and 75 to the indifference curve containing bundle D? Is the utility function you derived homogeneous?*

Answer: The function $u(x_1,x_2) = x_1^{0.5}x_2^{0.5} + 50$ would work. This function is not homogeneous (but it is homothetic).

(i) True or False: *Homothetic tastes can always be represented by functions that are homogeneous of degree k (where k is greater than zero), but even functions that are not homogeneous can represent tastes that are homothetic.*

Answer: This is true. We showed in the text that $MRS(tx_1,tx_2) = MRS(x_1,x_2)$ for homogeneous functions — thus, for homogeneous functions, the MRS is constant along any ray from the origin, the definition of homothetic tastes. At the same time, we just saw in the answer to the previous part an example of a non-homogeneous function that still represents homothetic tastes.

(j) True or False*: The marginal rate of substitution is homogeneous of degree 0 if and only if the underlying tastes are homothetic.*

Answer: For any set of homothetic tastes, the MRS is constant along rays from the origin; i.e. $MRS(tx_1,tx_2) = MRS(x_1,x_2)$. Thus, for homothetic tastes, the MRS is indeed homogeneous of degree 0. But $MRS(tx_1,tx_2) = MRS(x_1,x_2)$ defines homotheticity — so non-homothetic tastes will not have this property, which implies their MRS is not homogeneous of degree zero. The statement is therefore true.

Conclusion: Potentially Helpful Reminders

1. Keep in mind the distinction between how the MRS changes along an indifference curve (which tells us about substitutability) and how the MRS changes across the indifference map (which leads to ideas like homotheticity and quasilinearity).

2. The idea of substitutability will become critical in Chapter 7 when we introduce substitution effects (which will depend only on the shape of an indifference curve). The ideas of homotheticity and quasiliearity become impor-

tant as we introduce income effects (in Chapter 7) — which will be measured across an indifference map (rather than along an indifference curve).

3. Extremes like perfect substitutes and perfect complements are useful to keep in mind because they make it easy to remember which way an indifference map looks if the goods are relatively more substitutable as opposed to relatively more complementary and vice versa.

4. Special cases like homothetic and quasilinear tastes will become useful borderline cases in Chapter 7 — with homothetic tastes being the borderline case between luxury goods and necessities, and with quasilinear tastes being the borderline case between normal and inferior goods. (These terms are defined in Chapter 7.)

5. The Chapter 5 Exploring Relationship LiveGraphs — in particular those highlighted at the beginning of this chapter — are very helpful in developing intuition about the connection between math and graphs. They also give a good sense of just how many different types of tastes we can model with the tools we have developed.