CHAPTER

# 6

# Doing the “Best” We Can

We have talked about objective economic *circumstances* (Chapters 2 and 3) and subjective consumer *tastes* (Chapters 4 and 5). Now we start talking about **choice** or **behavior**. The choices people make, and the behavior we thus observe, results from people “doing the best they can given their circumstances” — where what is “best” depends not only on what choices are available (i.e. the person’s circumstances) but also how the person feels about the available alternatives (i.e. the person’s tastes). So we have to put together what we learned about economic circumstances with what we learned about tastes to talk about choice and behavior.

## Chapter Highlights

The main points of the chapter are:

1. When it is optimal for a consumer to choose some of each of the goods we are modeling, then the **marginal rate of substitution must be equal to the ratio of prices** at the consumer’s optimal bundle. If the *MRS* is not equal to the price ratio at the optimal bundle, then the consumer is at a **corner solution** and does not consume any of at least one of the goods.

2. When all consumers face the same prices, **all gains from trade are exhausted** without consumers having to trade with one another.

3. When all consumers face the same prices and end up choosing an interior optimum, they have the **same tastes at the margin after they optimize** even if their tastes are otherwise very different (and even if their incomes are very different.)

4. So long as all goods are “essential” (as defined in Chapter 5), the optimum bundle for every consumer will be an **interior optimum** where the consumer chooses some of all goods. **Multiple optimal bundles** can arise from **non-convexities** in either tastes or budgets.

5. **Behavior** is what we observe individuals actually doing. It results from individuals "doing the best they can given their circumstances" — i.e. it results from individuals combining tastes with budgets as they optimize. While tastes cannot be directly observed, we can infer something about underlying tastes from the actual behavior we observe.

# Using the LiveGraphs

For an overview of what is contained on the LiveGraphs site for each of the chapters (from Chapter 2 through 29) and how you might utilize this resource, see pages 2-3 of Chapter 1 of this *Study Guide*. To access the LiveGraphs for Chapter 6, click the *Chapter 6* tab on the left side of the LiveGraphs web site.

At this point, the LiveGraphs site for this chapter has mainly the typical *Animated Graphics*, *Static Graphics* and *Downloads*. Until we develop further material, there is one **Exploring Relationships** module that, like the modules in Chapter 5, is geared toward connecting math and intuition by allowing you to see how the consumer optimum changes for different parameters in budgets and tastes. While this involves the calculus-based material from part B of the *Microeconomics: An Intuitive Approach with Calculus* text, it is easy to operate the module without understanding all the underlying calculus. For this reason, we have made it available also on the LiveGraphs site for the non-calculus based *Microeconomics: An Intuitive Approach*.

# 6A Solutions to Within-Chapter-Exercises for Part A

**Exercise 6A.1** *In Chapter 2 we discussed a scenario under which my wife gives me a coupon that reduces the effective price of pants to $10 a pair. Assuming the same tastes, what would be my best bundle?*

Answer: In that case, the slope of the budget constraint is $-p_1/p_2 = -1$ — so the optimal bundle would have to have $MRS = -1$ as well. In describing tastes here, we said that the $MRS$ is equal to $-1$ at bundles where I have an equal number of shirts and pants — that is, along the 45 degree line. Thus, the optimal bundle would occur at the midpoint of the budget line that has intercepts of 20 on each axis — which is at the bundle (10,10) — 10 pants and 10 shirts.

**Exercise 6A.2** *Suppose both you and I have a bundle of 6 pants and 6 shirts, and suppose that my $MRS$ of shirts for pants is $-1$ and yours is $-2$. Suppose further that neither one of us has access to Wal-Mart. Propose a trade that would make both of us better off.*

Answer: In this case, you are willing to trade 2 shirts for 1 pair of pants whereas I am willing to trade them one for one. Assuming we can trade fractions of shirts and pants, a trade in which you give me 1.5 shirts for 1 pair of pants would make you better off (because you would have been willing to give up as many as 2 shirts for 1 pair of pants) and would also make me better off (because I would have been willing to accept as little as 1 shirt for 1 pair of pants). If we don't want to assume we can trade in fractions of goods, then the trade of 3 shirts for 2 pants would work similarly.

**Exercise 6A.3** *We keep using the phrase "at the margin" — as, for example, when we say that tastes for those leaving Wal-Mart will be the "same at the margin." What do economists mean by this "at the margin" phrase?*

Answer: "At the margin" means approximately around the bundle that we are discussing. To say that tastes are the same "at the margin" is the same as saying that around the bundles that individuals currently have (as they leave Wal-Mart), their tastes are the same — but that's not necessarily the same as saying that tastes are the same everywhere. "At the margin" restricts our attention to just a small subset of the larger space in which tastes reside.

**Exercise 6A.4** *In the previous section, we argued that Wal-Mart's policy of charging the same price to all consumers insures that there are no further gains from trade for goods contained in the shopping baskets of individuals that leave Wal-Mart. The argument assumed that all consumers end up at an interior solution, not a corner solution. Can you see why the conclusion still stands when some people optimize at corner solutions where their $MRS$ may be quite different from the $MRS$'s of those who optimize at interior solutions?*

Answer: When everyone optimizes at an interior solution, everyone's $MRS$ must be the same as everyone else's when they leave Wal-Mart — i.e. our tastes are the same at the margin, thus allowing for no further gains from trade. Now imagine that we consider shirts and pants — and someone leaves Wal-Mart with only shirts and no pants. That person, call her person A, is therefore at a corner solution — and for that corner solution to be optimal, it is almost certainly the case that this person's indifference curve is steeper than the budget constraint at the corner optimum. Thus, this person's tastes are not the same at the margin as those of the other consumers who optimized at a point where the slope of their budget constraint was equal to the slope of their indifference curve. Suppose, then, that person A's $MRS$ is $-4$ and person B's $MRS$ is $-2$ — with person B at an interior solution and person A at a corner solution where she buys only pants. Just looking at the $MRS$'s of the two people, we could say that a trade in which person A gives up 3 shirts in exchange for one pair of pants from person B would make both better off. After all, person B is willing to accept as few as 2 shirts for a pair of pants but would now get 3 instead, and person A is willing to give up as many as 4 shirts for a pair of pants but, under this trade, would only have to give up 3. The problem, however, is that person A has only pants — and therefore has not shirts to give up in a trade. Since person A's $MRS$ is higher in absolute value than person B's (and since this has to be the case in order for person A to be at a corner solution with only pants when person B is at an interior solution), the only potential trades that benefit both are those that have shirts going from A to B — but none of those trades is possible because A is at a corner solution and therefore without shirts to give up. Thus, when A and B leave Wal-Mart, there are no further gains from trade even if one (or both) of them is at a corner solution and their tastes are not the same at the margin. Either people who leave Wal-Mart are at an interior solution — in which case they have the same tastes on the margin as everyone else who is at an interior solution and thus can't trade with each other anymore; OR people are at a corner solution and don't have the same tastes as others on the margin but can't trade with them because they already have traded away every unit of the thing they value less at the margin than others who are at an interior solution. Either way, all gains from trade are exhausted in Wal-Mart — and the distribution of goods for people leaving Wal-Mart is efficient.

**Exercise 6A.5** *Suppose the prices of Coke and Pepsi were the same. Illustrate that now there are many optimal bundles for someone with my kind of tastes. What would be my "best" bundle if Pepsi is cheaper than Coke?*

Answer: When the prices of Coke and Pepsi are the same, then the budget constraint has the same slope as all the indifference curves. Therefore, one indifference curve lies right on top of the budget constraint and is therefore "tangent" at every point on the budget constraint. In that case, all bundles on the budget constraint are optimal bundles for the consumer. This makes intuitive sense — if Coke and Pepsi are priced the same and if I can't tell the difference between the two, it doesn't matter how I allocate my spending across Coke and Pepsi.

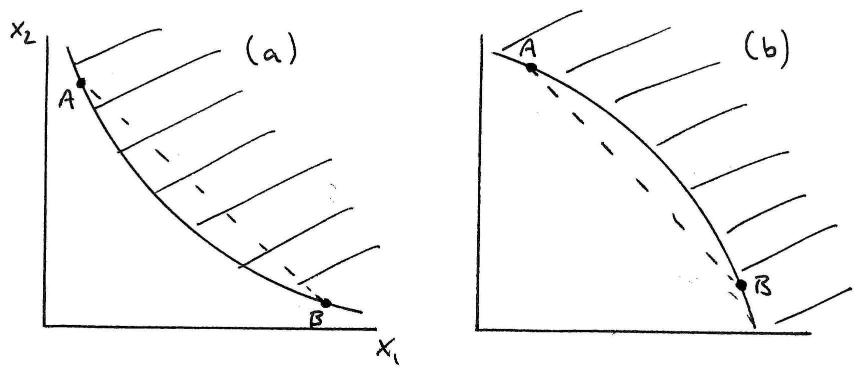
**Exercise 6A.6** *Consider a set of points that compose a solid sphere. Is this set convex? What about the set of points contained in a donut?*

Answer: Any line connecting two points in a solid sphere must necessarily be entirely contained within the sphere. Thus, a solid sphere is a convex set. If I pick two points on opposite sides of a donut, on the other hand, the line connecting them will lie (at least partially) outside the donut as it passes through the hole in the middle of the donut. Thus, a donut is not a convex set.

**Exercise 6A.7** *We have just defined what it means for a set of points to be convex — it must be the case that any line connecting two points in the set is fully contained in the set as well. In Chapter 4, we defined tastes to be convex when "averages are better than (or at least as good as) extremes". The reason such tastes are called "convex" is because the set of bundles that is better than any given bundle is a convex set. Illustrate that this is the case with an indifference curve from an indifference map of convex tastes.*

Answer: Panels (a) and (b) of Graph 6.1 (on the next page) illustrate two indifference curves, one from a map in which indifference curves satisfy the convexity property, and one from a map of indifference curves that does not satisfy convexity.

In both, the set of "better' bundles is shaded. Two bundles, $A$ and $B$, on each indifference curve are chosen and the line connecting them is indicated. That line lies fully in the shaded "better than" set in panel (a) but fully outside the shaded "better than" set in panel (b). Thus, convexity of tastes implies convex "better than" sets for each indifference curves, while non-convexities in tastes imply non-convex "better than" sets for some indifference curves.




Graph 6.1: Convexity and Tastes

**Exercise 6A.8** True/False*: If a choice set is non-convex, there are definitely multiple "best" bundles for a consumer whose tastes satisfy the usual assumptions.*

Answer: False. Non-convexities in choice sets imply that there *might* be multiple best bundles, not that there necessarily are for any given tastes of a consumer. In other words, it is easy to construct an indifference curve that only has one tangency on a non-convex budget constraint, but it is also possible to construct an indifference curve (that satisfies the convexity of tastes property) which has more than one tangency on a non-convex budget constraint.

**Exercise 6A.9** True/False*: If a choice set is convex, then there will be a unique "best" bundle assuming consumer tastes satisfy our usual assumptions and averages are strictly better than extremes.*

Answer: This is true. A convex choice set either bends out from the origin or is a straight line with negative slope and positive intercepts. A strictly convex indifference curve, on the other hand, bends toward the origin. Thus, as we move out to higher indifference curves, there will come a point where the budge constraint (that forms the boundary of a convex choice set) contains a single point in common with the indifference curve (that forms a convex "better than" set.)

**Exercise 6A.10** *Suppose that the choice set is defined by linear budget constraint and tastes satisfy the usual assumptions but contain indifference curves with linear components (or "flat spots"). True/False: Then there might be multiple "best" bundles but we can be sure that the set of "best" bundles is a convex set.*

Answer: True. When indifference curves have "flat spots", there is the potential that the line segment of the indifference curve (i.e. the "flat spot") has the same slope as the budget constraint and therefore each bundle on that segment is optimal (much like all bundles are optimal in the case of perfect substitutes when prices were the same for Coke and Pepsi in exercise 6A.5). The set of optimal bundles is then a line segment. Take any two points on the line segment, and it has to be the case that all points that lie on the line (between the points) connecting them also lies in the set of optimal bundles. Thus, the set of optimal bundles is itself a convex set. Of course it might also be the case that, with such indifference curves, the optimal bundle does not occur on the flat spot — and is therefore just a single point. But a set composed of a single point is trivially also a convex set.

**Exercise 6A.11** *True/False: When there are multiple "best" bundles due to non-convexities in tastes, the set of "best" bundles is also non-convex (assuming convex choice sets).*

Answer: True. When there are non-convexities in tastes, that means that the indifference curves at some point bend away from the origin. If multiple optimal bundles arise from that, it means that these bundles will not be connected as in the previous exercise — which means that the line connecting them will contain bundles that are not optimal. Thus, the set of optimal bundles is then non-convex.

# 6B Solutions to Within-Chapter-Exercises for Part B

**Exercise 6B.1** *Solve for the optimal quantities of $x_1$, $x_2$ and $x_3$ in the problem defined in equation 6.11. (Hint: The problem will be considerably easier to solve if you take the logarithm the utility function (which you can do since logarithms are order preserving transformations that do not alter the shapes of indifference curves.))*

Answer: Taking the hint in the problem, we can write the utility function as $v(x_1, x_2, x_3) = 0.5\ln x_1 + 0.5\ln x_2 + 0.5\ln x_3$ and the corresponding Lagrange function as

$$\mathcal{L}(x_1, x_2, x_3, \lambda) = 0.5\ln x_1 + 0.5\ln x_2 + 0.5\ln x_3 + \lambda(200 - 20x_1 - 10x_2 - 5x_3). \quad (6.1)$$

Taking first order conditions with respect to each good, we get

$$\begin{aligned} 0.5x_1^{-1} &= 20\lambda \\ 0.5x_2^{-1} &= 10\lambda \\ 0.5x_3^{-1} &= 5\lambda \end{aligned} \tag{6.2}$$

Dividing the first equation by the second and solving for $x_2$, we get $x_2 = 2x_1$. Dividing the first equation by the third and solving for $x_3$ we get $x_3 = 4x_1$. Substituting these into the budget constraint (which is the fourth first order condition taken with respect to $\lambda$), we get

$$20x_1 + 10(2x_1) + 5(4x_1) = 60x_1 = 200, \tag{6.3}$$

which implies $x_1 = 3.33$. Then, using the fact that $x_2 = 2x_1$ and $x_3 = 4x_1$, we get $x_2 = 6.67$ and $x_3 = 13.33$.

**Exercise 6B.2** *Set up the Lagrange function for this problem and solve it to see whether you get the same solution.*

Answer: The Lagrange function is

$$\mathcal{L}(x_1, x_2, \lambda) = \alpha \ln x_1 + x_2 + \lambda(200 - 20x_1 - 10x_2). \tag{6.4}$$

Taking first order conditions with respect to each variable in the Lagrange function, we get

$$\begin{aligned} &\frac{\alpha}{x_1} - 20\lambda = 0 \\ &1 - 10\lambda = 0 \\ &200 - 20x_1 - 10x_2 = 0 \end{aligned} \tag{6.5}$$

The second equation implies that $\lambda = 1/10$. Substituting this into the first equation, we get $x_1 = \alpha/2$, and substituting this into the last equation, we get $x_2 = (200 - 10\alpha)/10$.

**Exercise 6B.3** *Demonstrate how the Lagrange method (or one of the related methods we introduced earlier in this chapter) fails even worse in the case of perfect substitutes. Can you explain what the Lagrange method is doing in this case?*

Answer: Consider the utility function $u(x_1, x_2) = x_1 + x_2$. The Lagrange function would then be

$$\mathcal{L}(x_1, x_2, \lambda) = x_1 + x_2 + \lambda(I - p_1x_1 - p_2x_2), \tag{6.6}$$

with the first two first order conditions of

$$\begin{aligned} 1 &= \lambda p_1 \\ 1 &= \lambda p_2. \end{aligned} \tag{6.7}$$

Dividing these, we would get $p_1/p_2 = 1$ or $p_1 = p_2$. But that makes no sense — the prices are taken as given by the consumer. So, suppose $p_1 = 1$ and $p_2 = 2$. The first order conditions would then give us the "result" that $p_1 = 1 = p_2 = 2$. The Lagrange method fails because, as we have seen in the intuitive section of the chapter, there generally are no interior solutions to the optimization problem for a consumer whose tastes treat the goods as perfect substitutes. Instead, the consumer simply consumes only the good that is cheaper. The only time there are interior solutions occurs when $p_1 = p_2$ (our "result" from the Lagrange method) — but in that case any bundle on the budget line is in fact optimal.

**Exercise 6B.4** *At what value for $\alpha$ will the Lagrange method correctly indicate an optimal consumption of zero shirts? Which of the panels of Graph 6.10 illustrates this?*

Answer: It would have to be the case that the $MRS$ is equal to $-p_1/p_2 = -2$ at $x_1 = 10$. The $MRS$ for the utility function $u(x_1, x_2) = \alpha \ln x_1 + x_2$ is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{\alpha/x_1}{1} = -\frac{\alpha}{x_1}. \tag{6.8}$$

Thus, when $\alpha$ is such that $-\alpha/10 = -2$, the $MRS$ at $x_1 = 10$ is exactly equal to the slope of the budget constraint. Solving for $\alpha$ we get $\alpha = 20$.

You can check that this is correct by solving the optimization problem with Lagrange function

$$\mathscr{L}(x_1, x_2, \lambda) = 20 \ln x_1 + x_2 + \lambda(200 - 20x_1 - 10x_2). \tag{6.9}$$

The first two first order conditions of this problem are

$$\begin{aligned} \frac{20}{x_1} &= 20\lambda \\ 1 &= 10\lambda. \end{aligned} \tag{6.10}$$

These solve to give us $x_1 = 10$ and, plugging this back into the budget constraint, $x_2 = 0$. This is exactly what is illustrated in panel (a) of Graph 6.10.

**Exercise 6B.5** *In the previous section, we concluded that the first order conditions of the Lagrange problem may be misleading when goods are not essential. Are these conditions either necessary or sufficient in that case?*

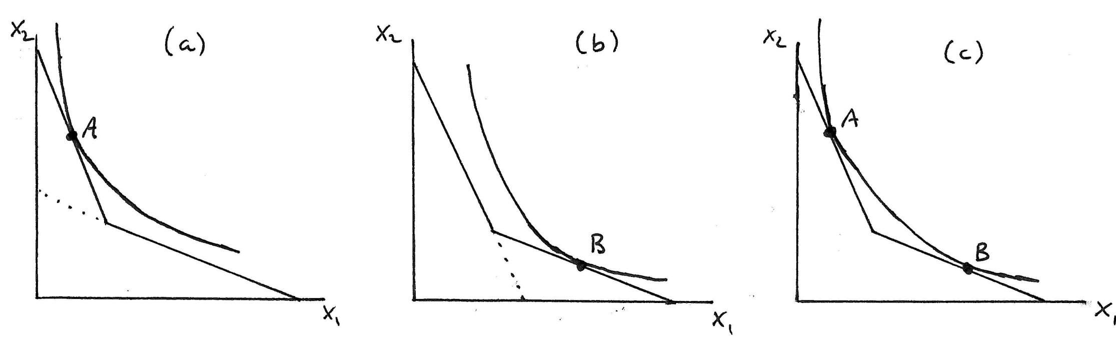
Answer: No. The conditions might not hold at the optimum (as we have seen in the case of corner solutions) — which means they are not necessary conditions for an optimum when goods are not essential. When they do hold, they might hold (as we have seen) at negative consumption levels when corner solutions are optimal — and so they are not sufficient. They are only sufficient for us to conclude we are at an optimum if they lead to positive consumption levels — in that case we would have an interior solution despite the fact that the goods are not essential.

**Exercise 6B.6** *Is it necessary for the indifference curve at the kink of the budget constraint to have a kink in order for both problems in (6.26) to result in* $x_1=6$*?*

Answer: No, it is not necessary so long as the kink points out rather than in. At the bundle (6,14), the indifference curve can have a slope between −2 and −1 and the kink point will in fact be optimal. (If the kink points in, however, then only an indifference curve that is also kinked at that bundle can result in this bundle being an optimum.)

**Exercise 6B.7** *Using the intuitions from graphical analysis similar to that in Graph 6.14, illustrate how you might go about solving for the true optimum when a choice set is non-convex due to an "inward" kink.*

Answer: Essentially, there are three different possibilities, depicted in panels (a) through (c) of Graph 6.2. In panel (a), the optimal bundle is clearly bundle *A* which in fact will be the solution to the Lagrange problem that uses the steeper budget line. The Lagrange problem that uses the shallower budget line might produce an "optimal" bundle that lies on the dashed portion of that shallower budget — in which case we know it can't be optimal given that the steeper budget contains bundles that have strictly more of everything. Alternatively, the Lagrange problem that uses the shallower budget might result in an "optimal" bundle that lies on the solid portion of that shallower budget — but when we determine the utility level at that bundle and compare it to *A* we would find the utility at *A* to be higher.




Graph 6.2: Optimization with an Inward Kink

In panel (b), the optimal bundle is *B* on the shallower portion of the budget constraint. In that case, the Lagrange problem that uses the shallower budget will find this optimal bundle. The Lagrange problem that uses the steeper budget might find an "optimal" bundle on the dashed portion of the steeper budget (in which case we would immediately know that it was not truly optimal since bundles with more of everything are in fact available) or on the solid portion. In the latter case, we we would compare the utility at that bundle to that from $B$ and find that the utility at $B$ is greater.

Finally, panel (c) illustrates the special case where the Lagrange problem with the steeper budget gives us $A$ as the optimal bundle and the Lagrange problem with the shallower budget gives us $B$ — and when we plug both of them back into the utility function, we find that they give the same utility. In that case, we have found two optimal bundles.

# End of Chapter Exercises

## Exercise 6.4: *Inferring Tastes for Roses (and Love) from Behavior*

*Inferring Tastes for Roses (and Love) from Behavior: I express my undying love for my wife through weekly purchases of roses that cost \$5 each.*

**A:** *Suppose you have known me for a long time and you have seen my economic circumstances change with time. For instance, you knew me in graduate school when I managed to have \$125 per week in disposable income that I could choose to allocate between purchases of roses and "other consumption" denominated in dollars. Every week I brought 25 roses home to my wife.*

(a) *Illustrate my budget as a graduate student — with roses on the horizontal and "dollars of other consumption" on the vertical axis. Indicate my optimal bundle on that budget as $A$. Can you conclude whether either good is not "essential"?*

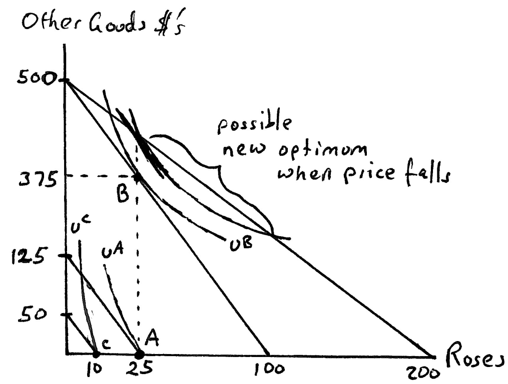
Answer: Graph 6.3 (on the next page) illustrates a number of different budget constraints for this problem, including the one described in this part — which starts at \$125 on the vertical axis and ends at 25 roses on the horizontal axis. Thus, I am spending all my income on roses — which implies other goods are not essential for me.

(b) *When I became an assistant professor, my disposable income rose to \$500 per week, and the roses I bought for my wife continued to sell for \$5 each. You observed that I still bought 25 roses each week. Illustrate my new budget constraint and optimal bundle $B$ on your graph. From this information, can you conclude whether my tastes might be quasilinear in roses? Might they not be quasilinear?*

Answer: The new budget constraint is the one starting at \$500 on the vertical axis and ending at 100 roses on the horizontal. The new optimal $B$ lies exactly above the original optimal $A$. These tastes could be quasilinear — it is possible that the $MRS$ at $A$ is exactly equal to the $MRS$ at $B$. But tastes might also not be quasilinear because the $MRS$ at $A$ could in fact be larger in absolute value (i.e. the slope could be steeper) at $A$ than at $B$ — which would still make the corner solution at $A$ optimal.

(c) *Suppose for the rest of the problem that my tastes in fact are quasilinear in roses. One day while I was an assistant professor, the price of roses suddenly dropped to \$2.50. Can you predict whether I then purchased more or fewer roses?*

Answer: The new budget line is the one beginning at \$500 on the vertical axis and ending at 200 roses on the horizontal. The bundle that lies on this budget line and directly above $B$ must have an $MRS$ that is the same as the $MRS$ that goes through $B$ if tastes are indeed quasilinear. But that implies that the indifference curve through that point cuts the budget line from above — making bundles to the right more preferred. Thus, I can conclude I will consume more roses when the price of roses falls.




Graph 6.3: Love and Roses

(d) *Suppose I had not gotten tenure — and the best I could do was rely on a weekly allowance of \$50 from my wife. Suppose further that the price of roses goes back up to \$5. How many roses will I buy for my wife per week?*

Answer: This budget constraint begins at \$50 on the vertical axis and ends at 10 roses on the horizontal. If tastes are indeed quasilinear, the *MRS* at the corner bundle *C* is larger in absolute value (i.e. the slope is steeper) than it is at *A* or *B*. Thus, if *A* was an optimum under the higher budget, *C* must be an optimum under the lower income. I will therefore buy 10 roses per week.

(e) True or False*: Consumption of quasilinear goods always stays the same as income changes.*

Answer: This is almost true but not quite. As we have shown, once we reach the corner solution where we are only consuming the quasilinear good, we will reduce our consumption of that good as income falls (because we just don't have enough income to keep buying the same amount).

(f) True or False*: Over the range of prices and incomes where corner solutions are not involved, a decrease in price will result in increased consumption of quasilinear goods but an increase in income will not.*

Answer: This is true. We have demonstrated in part (c) that decreases in prices will lead to increased consumption of the quasilinear good. We also know that, for quasilinear goods, the *MRS* stays constant along any consumption level of the other good — which implies that the tangency of the budget line and the optimal indifference curve remains at the same level of the quasilinear good as income increases (because increases in income do not change the slope of budget constraints and thus don't change the slope of the optimal indifference curve at the optimum so long as we are not at corner solutions).

**B:** *Suppose my tastes for roses ($x_1$) and other goods ($x_2$) can be represented by utility function* $u(x_1, x_2) = \beta x_1^\alpha + x_2$.

(a) *Letting the price of roses be denoted by $p_1$, the price of other goods by 1, and my weekly income by I, determine my optimal weekly consumption of roses and other goods as a function of $p_1$ and I.*

Answer: The Lagrange function for this optimization problem is

$$\mathcal{L}(x_1, x_2, \lambda) = \beta x_1^\alpha + x_2 + \lambda(I - p_1 x_1 - x_2). \tag{6.11}$$

The first two first order conditions are

$$\begin{aligned} \alpha\beta x_1^{\alpha-1} &= \lambda p_1 \\ 1 &= \lambda. \end{aligned} \tag{6.12}$$

Replacing $\lambda$ with 1 in the first equation and solving for $x_1$, we get

$$x_1 = \left(\frac{p_1}{\alpha\beta}\right)^{1/(\alpha-1)} = \left(\frac{\alpha\beta}{p_1}\right)^{1/(1-\alpha)} \tag{6.13}$$

and substituting this into the budget constraint and solving for $x_2$, we get

$$x_2 = I - p_1\left(\frac{\alpha\beta}{p_1}\right)^{1/(1-\alpha)} = I - \frac{\alpha\beta}{p_1^{\alpha}}^{1/(1-\alpha)}. \tag{6.14}$$

Note that $x_1$ is not a function of $I$ — the optimal level of the quasilinear good is independent of $I$ (so long as the Lagrange method applies — i.e. so long as we are not at a corner solution).

(b) *Suppose $\beta = 50$ and $\alpha = 0.5$. How many roses do I purchase when $I = 125$ and $p_1 = 5$? What if my income rises to \$500?*

Answer: Substituting $\beta = 50$, $\alpha = 0.5$, $I = 125$ and $p_1 = 5$ into equations (6.13) and (6.14), we get $x_1 = 25$ and $x_2 = 0$, exactly like point $A$ in our graph. When we replace income by \$500, we get $x_1 = 25$ and $x_2 = 375$, again exactly as in our graph.

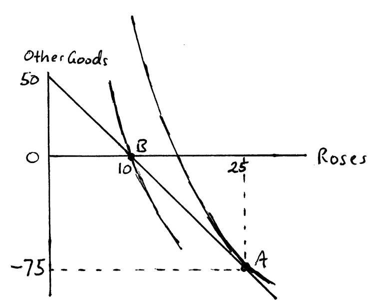
(c) *Comparing your answers to your graph from part A, could the actions observed in part A(b) be rationalized by tastes represented by the utility function $u(x_1, x_2)$? Give an example of another utility function that can rationalize the behavior described in part A(b).*

Answer: Yes, as we just showed, the utility function gives us the same optimal consumption levels as those graphed in Graph 6.3. Any order preserving transformation of the utility function will similarly rationalize this behavior — as, for instance, $v(x_1, x_2) = \left(\beta x_1^{\alpha} + x_2\right)^2$.

(d) *What happens when the price of roses falls to \$2.50? Is this consistent with your answer to part A(c)?*

Substituting $\beta = 50$, $\alpha = 0.5$, $I = 500$ and $p_1 = 2.5$ into equations (6.13) and (6.14), we get $x_1 = 100$ and $x_2 = 250$ — which is consistent with the answer we gave in A(c), i.e. the answer that I will buy more roses when the price falls.

(e) *What happens when my income falls to \$50 and the price of roses increases back to \$5? Is this consistent with your answer to part A(d)? Can you illustrate in a graph how the math is giving an answer that is incorrect?*




Graph 6.4: Love and Roses: Part 2

Answer: Substituting $\beta = 50$, $\alpha = 0.5$, $I = 50$ and $p_1 = 5$ into equations (6.13) and (6.14), we get $x_1 = 25$ and $x_2 = -75$. This can't be a true optimum because it would involve a negative consumption level for other goods. Graph 6.4 illustrates what is happening — that math picks out bundle $A$ in the graph because that is where an indifference curve extended into the negative "other goods" quadrant is tangent to a budget line that is similarly extended. The negative value for "other goods" suggests that there is a corner solution that is missed by the math because there is no tangency at that solution. This solution, we know from our intuition, is the bundle $B$=(10,0). The indifference curve through that bundle has a steeper slope than the budget constraint as we can see by calculating the $MRS$ for this utility function. Applying our formula for $MRS$, we get $MRS = -\alpha\beta x_1^{\alpha-1}$ which, when $\alpha = 0.5$, $\beta = 50$ and $x_1 = 10$ is substituted into it, implies that the $MRS$ at $x_1 = 10$ is $-7.91$ while we know the budget constraint has a slope of just $-5$. Thus, the answer given by the math is just wrong because there is a corner solution, but the corner solution it points us to is exactly the one we arrived at in part A(d).

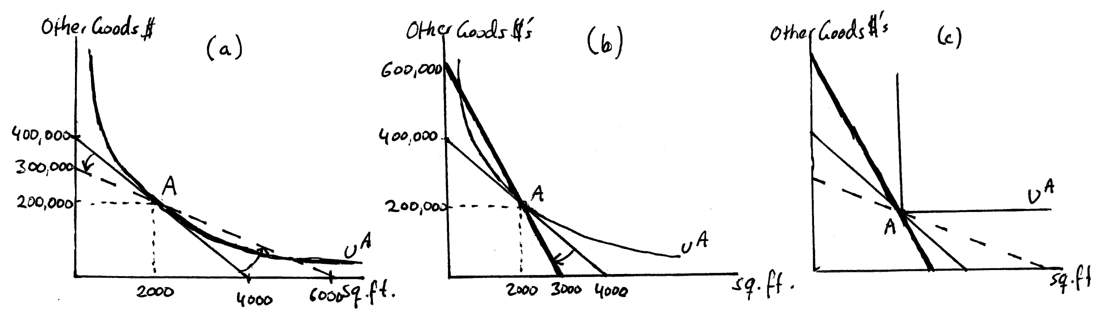
## Exercise 6.9: *Price Fluctuations in the Housing Market*

Everyday Application: *Price Fluctuations in the Housing Market: Suppose you have $400,000 to spend on a house and "other goods" (denominated in dollars).*

**A:** *The price of 1 square foot of housing is $100 and you choose to purchase your optimally sized house at 2000 square feet. Assume throughout that you spend money on housing solely for its consumption value (and not as part of your investment strategy).*

(a) *On a graph with "square feet of housing" on the horizontal axis and "other goods" on the vertical, illustrate your budget constraint and your optimal bundle $A$.*

Answer: The budget constraint would have vertical intercept of $400,000 (since this is how much other goods you can consume if you buy no housing) and horizontal intercept of $4,000 square feet of housing (since that is how much you can afford at $100 per square foot if you spend all your money on housing.) The slope of this budget is -100. The budget is depicted as the solid line in panel (a) of Graph 6.5.




Graph 6.5: Housing Price Fluctuations

(b) *After you bought the house, the price of housing falls to $50 per square foot. Given that you can sell your house from bundle A if you want to, are you better or worse off?*

Answer: The (dashed) new budget line is also drawn in panel (a) of the graph. Note that it has to go through $A$ because $A$ is your endowment point once you have bought the 2000 square foot house. Thus, you can always choose to consume that bundle regardless of what happens to prices. But you can also sell your 2000 square foot house for $100,000 — which would give you $300,000 in consumption, your new vertical intercept. Or you can take that $300,000 and spend it on a new house and thereby buy as much as a 6,000 square foot house since housing now only costs $50 per square foot. Since your indifference curve at $A$ is tangent to your original budget line, the new (shallower) budget line cuts that indifference curve from below at bundle $A$. All the new bundles that are now affordable and that lie above the original indifference curve $u^A$ therefore lie to the right of $A$. You are better off at any of those bundles on the dashed line that lie above the indifference curve $u^A$.

(c) *Assuming you can easily buy and sell houses, will you now buy a different house? If so, is your new house smaller or larger than your initial house?*

Answer: You will buy a larger house — since all the better bundles on the dashed line in panel (a) are to the right of $A$ and therefore include a house larger than 2000 square feet.

(d) *Does your answer to (c) differ depending on whether you assume tastes are quasilinear in housing or homothetic?*

Answer: No — in both cases you would end up better off consuming a larger house.

(e) *How does your answer to (c) change if the price of housing went up to $200 per square foot rather than down to $50.*

Answer: Panel (b) of Graph 6.5 illustrates this change in prices. The original budget constraint (from $400,000 on the vertical to 4,000 square feet on the horizontal axis) with bundle $A$ is replicated from panel (a) and illustrates the budget when the price per square foot of housing is $100. The steeper bold line going through $A$ illustrates the new budget line when $A$ is the endowment point and the price of housing goes to $200 per square foot. If you sell your 2000 square foot house at $200 per square foot, you would get $400,000 for it — which, added to the $200,000 you have would give you as much as $600,000 in consumption if you choose not to buy another house. If you do buy another house, the largest possible house at the new prices is now a 3000 square foot house. But you can always choose to stay at $A$ — so $A$ too is on the new budget line. The bundles on the new bold budget that also lie above the indifference curve $u^A$ all lie to the left of $A$ — indicating that the new house that you would purchase would be smaller than your original 2000 square foot house.

(f) *What form would tastes have to take in order for you to not sell your $2000 square foot house when the price per square foot goes up or down?*

Answer: The indifference curve through $A$ would have to have a kink in it, as would be the case if housing and other goods are perfect complements. This is illustrated in panel (c) of Graph 6.5 where all three budget lines are drawn, as is an indifference curve $u^A$ that treats the two goods as perfect complements. Technically, it could also be the case that the indifference curve through $A$ has a less severe kink at $A$ — one where the slope to the left of $A$ is steeper than the bold budget line and the slope to the right of $A$ is shallower than the slope of the dashed budget line. What is important is that there is a sufficiently severe kink — with no substitutability on the margin between the goods at the kink point. If there is no kink at $A$ — i.e. if there is any substitutability at the margin between housing and other goods at $A$ — then the bold and dashed indifference curves must necessarily cut the indifference curve at $A$ in the ways (though not necessarily with the magnitudes) illustrated in (a) and (b).

(g) True or False*: So long as housing and other consumption is at least somewhat substitutable, any change in the price per square foot of housing makes homeowners better off (assuming it is easy to buy and sell houses.)*

Answer: This is true, as just argued in the answer above.

(h) True or False*: Renters are always better off when the rental price of housing goes down and worse off when it goes up.*

Answer: This is true. Renters do not have endowment points in this model as homeowners do. So changes in the rental price of housing rotate the budget line through the vertical intercept — which implies that a drop in housing prices unambiguously expands the budget set at every level of housing and an increase in housing prices unambiguously shrinks the choice set at every level of housing.

**B:** *Suppose your tastes for "square feet of housing" ($x_1$) and "other goods" ($x_2$) can be represented by the utility function $u(x_1, x_2) = x_1 x_2$.*

(a) *Calculate your optimal housing consumption as a function of the price of housing ($p_1$) and your exogenous income $I$ (assuming of course that $p_2$ is by definition equal to 1.)*

Answer: We want to solve the problem

$$\max_{x_1,x_2} u(x_1,x_2) = x_1x_2 \text{ subject to } p_1x_1 + x_2 = I. \tag{6.15}$$

The Lagrange function for this problem is

$$\mathcal{L}(x_1,x_2,\lambda) = x_1x_2 + \lambda(I - p_1x_1 - x_2), \tag{6.16}$$

which give us first order conditions

$$\begin{aligned} x_2 &= \lambda p_1 \\ x_1 &= \lambda \\ p_1x_1 + x_2 &= I. \end{aligned} \tag{6.17}$$

Substituting the second equation into the first, we get $x_2 = x_1p_1$, and substituting this into the last equation, we get $p_1x_1 + p_1x_1 = I$ or $x_1 = I/(2p_1)$. Finally, plugging this back into $x_2 = x_1p_1$, we get $x_2 = I/2$.

(b) *Using your answer, verify that you will purchase a 2000 square foot house when your income is \$400,000 and the price per square foot is \$100.*

Answer: We just concluded that $x_1 = I/(2p_1)$. When $p_1 = 100$ and $I = 400,000$, this implies $x_1 = 400,000/(2(100)) = 2000$.

(c) *Now suppose the price of housing falls to \$50 per square foot and you choose to sell your 2000 square foot house. How big a house would you now buy?*

Answer: By selling your 2000 square foot house at \$50 per square foot, you would make \$100,000. Added to the \$200,000 you had left over after you bought your original 2000 square foot house, this gives you a total income of \$300,000. Plugging $I$=300,000 and $p_1 = 50$ into our equation for the optimal housing quantity $x_1 = I/(2p_1)$, we get $x_1$=300,000/(2(50))=3000. Thus, you will buy a 3000 square foot house.

(d) *Calculate your utility (as measured by your utility function) at your initial 2000 square foot house and your new utility after you bought your new house? Did the price decline make you better off?*

Answer: Your initial consumption bundle was (2000, 200000). That gives utility

$$u(2000,200000) = 2000(200000) = 400,000,000. \tag{6.18}$$

When price fell, you end up at the bundle (3000,150000) which gives utility

$$u(3000,150000) = 3000(150000) = 450,000,000. \tag{6.19}$$

Since your utility after the price decline is higher than before, you are better off.

(e) *How would your answers to B(c) and B(d) change if, instead of falling, the price of housing had increased to \$200 per square foot?*

Answer: Again, we have already calculated that $x_1 = I/(2p_1)$ and $x_2 = I/2$. When price increases to \$200 and you already own a 2000 square foot house, you can now sell your house for \$400,000 which, added to the \$200,000 you had left over after buying your original house, gives you up to \$600,000 to spend. Treating this as your new $I$ and plugging in the new housing price $p_1 = 200$, we then get that your new optimal bundle has $x_1 = 600000/(2(200)) = 1500$ and $x_2 = 600000/2 = 300,000$. Thus you will buy a 1500 square foot house and consume \$300,000 in other goods. This gives you utility

$$u(1500,300000) = 1500(300000) = 450,000,000, \tag{6.20}$$

which is greater than the utility you had originally and equal to the utility you received from the price decrease above. Thus, a price increase to \$200 per square foot makes you better off, exactly as much as a drop in price to \$50 per square foot. You are therefore indifferent between the price increase and the price decrease.

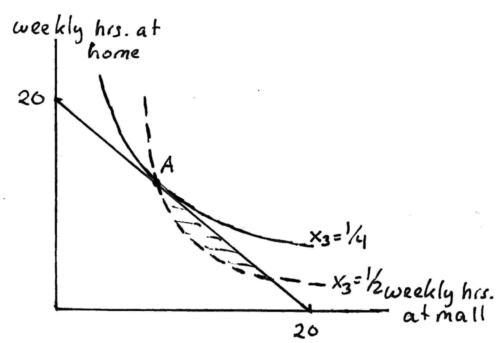
# Exercise 6.12: *Retail Industry Lobbying for Daylight Savings Time*

Business Application: *Retail Industry Lobbying for Daylight Savings Time: In 2005, the U.S. Congress passed a bill to extend daylight savings time earlier into the spring and later into the fall (beginning in 2007). The change was made as part of an Energy Bill, with some claiming that daylight savings time reduces energy use by extending sunlight to later in the day (which requires fewer hours of artificial light). Among the biggest advocates for daylight savings time, however, was the retail and restaurant industry that believes consumers will spend more time shopping and eating in malls for reasons explored here.*

**A:** *Consider a consumer who returns home from work at 6PM and goes to sleep at 10PM. In the month of March, the sun sets by 7PM in the absence of daylight savings time, but with daylight savings time, the sun does not set until 8PM. When the consumer comes home from work, she can either spend time (1) at home eating food from her refrigerator while e-mailing friends and surfing/shopping on the internet or (2) at the local mall meeting friends for a bite to eat and strolling through stores to shop. Suppose this consumer gets utility from (1) and (2) (as defined here) but she also cares about $x_3$ which is defined as the fraction of daylight hours after work.*

(a) *On a graph with "weekly hours at the mall" on the horizontal axis and "weekly hours at home" on the vertical, illustrate this consumer's typical weekly after-work time constraint (with a total of 20 hours per week available — 4 hours on each of the 5 workdays). (For purposes of this problem, assume the consumer gets as much enjoyment from driving to the mall as she does being at the mall).*

Answer: This is illustrated in Graph 6.6. The consumer can spend either 20 hours at home or 20 hours at the mall or some combination of 20 hours between the two places. Thus, the opportunity cost of spending 1 hour at the mall is not being able to spend that hour at home — leading to a slope of $-1$.




Graph 6.6: Daylight Savings Time

(b) *Consider first the scenario of no daylight savings time in March. This implies only 1 hour of daylight in the 4 hours after work and before going to sleep; i.e. the fraction $x_3$ of daylight hours after work is 1/4. Pick a bundle A on the budget constraint from (a) as the optimum for this consumer given this fraction of after-work of daylight hours.*

Answer: This is also depicted in the graph, with the (solid) indifference curve going through $A$ labeled by $x_3 = 1/4$ and tangent to the budget constraint.

(c) *Now suppose daylight savings time is moved into March, thus raising the number of after-work daylight hours to 2 per day. Suppose this changes the MRS at every bundle. If the retail and restaurant industry is right, which way does it change the $MRS$?*

Answer: If the industry is right, then more sunlight leads consumers to, at every bundle, be willing to give up more hours at home for an hour at the mall. Thus, the $MRS$ becomes larger in absolute value — leading to indifference curves with steeper slopes.

(d) *Illustrate how, if the retail and restaurant industry is right, this results in more shopping and eating at malls every week.*

Answer: This is illustrated with the second (dashed) indifference curve in the graph — with that indifference curve also passing through $A$ but now labeled $x_3 = 1/2$. This indifference curve has steeper slope as we concluded it must have if the retail and restaurant industry is right. The shaded bundles between the new indifference curve and the budget line all make the consumer better off than bundle $A$ when $x_3 = 1/2$ — and each of these bundles involves more time spent at the mall and restaurants.

(e) *Explain the following statement: "While it appears in our 2-dimensional indifference maps that tastes have changed as a result of a change in daylight savings time, tastes really haven't changed at all because we are simply graphing 2-dimensional slices of the same 3-dimensional indifference surfaces."*

Answer: The consumer's tastes have really not changed in any fundamental way — the consumer always cared about sunlight but simply does not have control over how much sunlight there is. Thus, for purposes of analyzing choices given the level of sunlight (i.e. given $x_3$), we only have to focus on the slice of the 3-dimendsional indifference surfaces that illustrate combinations of mall hours, home hours and sunlight that the consumer is indifferent between. When daylight savings time goes into effect, sunlight changes and the consumer switches to a different portion of the 3-dimensional indifference surface — the portion that is now relevant for the new level of sunlight. When these two slices of the 3-dimensional surfaces are depicted on a single graph, it looks like indifference curves cross and tastes must have changed — but that is only because we are projecting two slices of the same tastes onto a single 2-dimensional graph.

(f) *Businesses can lobby Congress to change the* circumstances *under which we make decisions, but Congress has no power to change our* tastes. *Explain how the change in daylight savings time illustrates this in light of your answer to (e).*

Answer: Congress did not need to change tastes in order to change behavior in line with what retailers and restaurant owners lobbied for — all it needed to do was change the circumstances consumers face — which in this case includes the number of hours of daylight after working hours. Often Congress changes individual circumstances through such policies as taxes or spending — but it also does so through regulations like when daylight savings time begins. As circumstances change, behavior changes even if tastes remain the same.

(g) *Some have argued that consumers must be irrational for shopping more just because daylight savings is introduced. Do you agree?*

Answer: Our model suggests there is nothing irrational at all about shopping more under daylight savings time. By extending daylight hours at the end of the day, Congress has made it more desirable to go shopping because people like to shop while it is still daylight out. As a result, people shop more — they are merely responding to changed circumstances but are optimizing given their circumstances both before and after daylight savings time goes into effect.

(h) *If we consider not just energy required to produce light but also energy required to power cars that take people to shopping malls, is it still clear that the change in daylight savings time is necessarily energy saving?*

Answer: No, it is not clear since there are offsetting effects that result from the change in people's behavior. In fact, there are studies that claim to show that daylight savings time actually costs more energy because of such adjustments in individual behavior to changed circumstances.

**B:** *Suppose a consumer's tastes can be represented by the utility function $u(x_1,x_2,x_3) = 12x_3 \ln x_1 + x_2$, where $x_1$ represents weekly hours spent at the mall, $x_2$ represents weekly after-work hours spent at home (not sleeping), and $x_3$ represents the fraction of after-work (before-sleep) time that has daylight.*

(a) *Calculate the $MRS$ of $x_2$ for $x_1$ for this utility function and check to see whether it has the property that retail and restaurant owners hypothesize.*

Answer: The $MRS$ is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{12x_3/x_1}{1} = -\frac{12x_3}{x_1}. \tag{6.21}$$

We concluded in part A that retail and restaurant owners believe that, if there is more daylight, people will be willing to give up more hours at home for every hour at the mall and restaurants. This implies that the $MRS$ should become larger in absolute value as $x_3$ — the fraction of evening time with daylight — increases. That is indeed the case — as $x_3$ increases, $12x_3/x_1$ increases as well — causing the indifference curves at every bundle to become steeper.

(b) *Which of the three things the consumer cares about — $x_1, x_2$ and $x_3$ — are choice variables for the consumer?*

Answer: The consumer only get to choose $x_1$ and $x_2$ — the amount of time spent outside and inside the house. She does not get to decide how much daylight there is in the rest of the day.

(c) *Given the overall number of weekly after-work hours our consumer has (i.e. 20), calculate the number of hours per week this consumer will spend in malls and restaurants as a function of $x_3$.*

Answer: The problem we have to solve is

$$\max_{x_1,x_2} u(x_1,x_2,x_3) = 12x_3 \ln x_1 + x_2 \text{ subject to } x_1 + x_2 = 20. \tag{6.22}$$

Note that, since $x_3$ is not a choice variable, it does not appear as part of the max notation in the specification of the problem. The Lagrange function for this problem is then

$$\mathscr{L}(x_1,x_2,\lambda) = 12x_3 \ln x_1 + x_2 + \lambda(20 - x_1 - x_2), \tag{6.23}$$

where the Lagrange function is not a function of $x_3$ because $x_3$ enters the function only as a parameter (exactly as the number "12" does), not as a variable. First order conditions are then taken only with respect to $x_1$ and $x_2$ (and $\lambda$), with the first two of these giving us

$$\begin{aligned} \frac{12x_3}{x_1} &= \lambda \\ 1 &= \lambda, \end{aligned} \tag{6.24}$$

where the second equation can just be substituted into the first to get $x_1 = 12x_3$. Notice that $x_1$ is only a functions of $x_3$ and not of $x_2$ — that's because tastes are quasilinear in $x_1$. (We can also derive the number of hours spent at home by simply putting $x_1 = 12x_3$ into the budget constraint $x_1 + x_2 = 20$ to get $12x_3 + x_2 = 20$ or $x_2 = 20 - 12x_3$.)

(d) *How much time per week will she spend in malls and restaurants in the absence of daily savings time? How does this change when daylight savings time is introduced?*

Answer: In the absence of daylight savings time, $x_3 = 1/4$ which implies $x_1 = 12x_3 = 3$. With daylight savings time, $x_3 = 1/2$ which implies $x_1 = 12x_3 = 6$. Thus, daylight savings time causes this consumer to go to the mall for 6 hours per week instead of 3 hours per week.

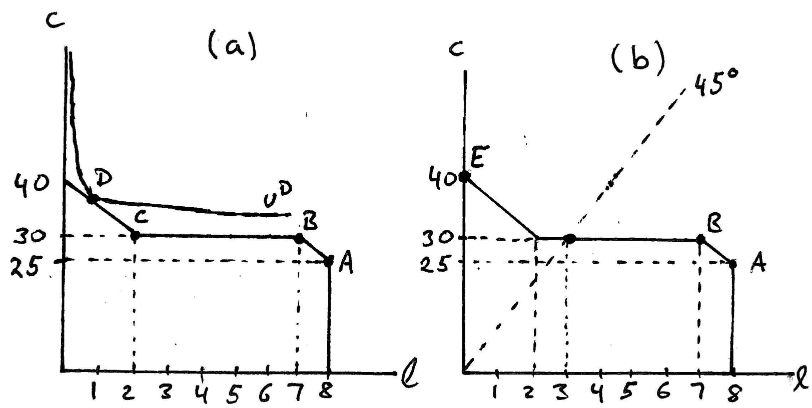
## Exercise 6.15: *AFDC and Work Disincentives*

Policy Application: *AFDC and Work Disincentives*: *Consider the AFDC program for an individual as described in end-of-chapter exercise 3.18.*

**A:** *Consider again an individual who can work up to 8 hours per day at a wage of $5 per hour.*

(a) *Replicate the budget constraint you were asked to illustrate in 3.18A.*

Answer: This is done in panel (a) of Graph 6.7 (on the next page), with leisure hours on the horizontal and consumption dollars on the vertical axis.




Graph 6.7: AFDC and Work Disincentives

(b) True or False*: If this person's tastes are homothetic, then he/she will work no more than 1 hour per day.*

Answer: This is false. Suppose, for instance, that leisure and consumption were perfect complements in the sense that this person wants to consume 1 hour of leisure with every \$35 of consumption. Indifference curves would then be L-shaped, with corners happening at bundles like (1,35) and (2,70). This would imply an optimal choice at (1,35) where the worker takes exactly 1 hour of leisure per day and works 7 hours per day. Such tastes are homothetic, as are less extreme tastes that allow for some (but not too much) substitutability between leisure and consumption. An example of an indifference curve $u^D$ from a somewhat less extreme indifference map is illustrated in panel (a) of the graph — with tangency at $D$.

(c) *For purposes of defining a 45-degree line for this part of the question, assume that you have drawn hours on the horizontal axis 10 times as large as dollars on the vertical. This implies that the 45-degree line contains bundles like (1,10), (2,20), etc. How much would this person work if his tastes are homothetic and symmetric across this 45-degree line? (By "symmetric across the 45-degree line" I mean that the portions of the indifference curves to one side of the 45 degree line are mirror images to the portions of the indifference curves to the other side of the 45 degree line.)*

Answer: Panel (b) of the graph depicts this "45 degree line" where \$10 on the vertical axis is the same distance as 1 hour on the horizontal. In order for indifference curves to be symmetric around this line, it must be that the slope of the indifference curve for bundles on the 45 degree line is $-1$. But since we are measuring \$10 as geometrically equivalent to 1 hour, a slope of $-1$ is really a slope, or $MRS$ of $-10$. If we were to draw a line from the point (0,40) to (3,30), this line would have a slope of $-10/3$. But any indifference curve has a slope of $-10$ on the 45 degree line — so we know that the indifference curve at (3,30) has a slope of $-10$ at that point and gets steeper to the left. So all indifference curves going through (3,30) or above on the 45 degree line pass above the budget constraint to the left of the 45 degree line. Thus, such "symmetric" tastes will have an optimum to the right of the 45 degree line — most likely at $B$ but plausibly between $B$ and $A$.

(d) *Suppose you knew that the individual's indifference curves were linear but you did not know the $MRS$. Which bundles on the budget constraint could in principle be optimal and for what ranges of the $MRS$?*

Answer: Bundles on the budget between $A$ and $B$ could be optimal, as could bundle $E$. In particular for $MRS$ between 0 and $-10/7$, $E$ would be optimal and the individual would work all the time and take no leisure. This is because indifference curves would be straight lines with sufficiently shallow slope to make the corner solution $E$ optimal. For $MRS$ between $-10/7$ and $-5$, $B$ would be optimal. For $MRS = -5$, any bundle on the budget between $B$ and $A$ is optimal, with all these bundles lying on one indifference curve that is also the highest possible indifference curve for such an individual. Finally, for $MRS$ less than $-5$, $A$ becomes the optimal bundle.

(e) *Suppose you knew that, for a particular person facing this budget constraint, there are two optimal solutions. How much in AFDC payments does this person collect at each of these optimal bundles (assuming the person's tastes satisfy our usual assumptions)?*

Answer: The only way there can be exactly two optimal solutions is if one of these is $B$ and the other lies anywhere from $E$ to $C$. The person collects no AFDC between $E$ and $C$ but the full \$25 daily benefit at $B$.

**B:** *Suppose this worker's tastes can be summarized by the Cobb-Douglas utility function $u(\ell, c) = \ell^{1-\alpha} c^{\alpha}$ where $\ell$ stands for leisure and $c$ for consumption.*

(a) *Forget for a moment the AFDC program and suppose that the budget constraint for our worker could simply be written as $c = I - 5\ell$. Calculate the optimal amount of consumption and leisure as a function of $\alpha$ and $I$.*

Answer: We need to solve the problem

$$\max_{\ell,c} u(\ell,c) = \ell^{1-\alpha} c^{\alpha} \text{ subject to } c = I - 5\ell. \tag{6.25}$$

Setting up the Lagrangian, taking first order conditions and solving for $\ell$ and $c$, we get

$$\ell = \frac{(1-\alpha)I}{5} \text{ and } c = \alpha I. \tag{6.26}$$

(b) *On your graph of the AFDC budget constraint for this worker, there are two line segments with slope $-5$ — one for 0-2 hours of leisure and another for 7-8 hours of leisure. Each of these lie on a line defined by $c = I - 5\ell$ except that $I$ is different for the two equations that contain these line segments. What are the relevant $I$'s to identify the right equations on which these budget constraint segments lie?*

Answer: It's easy to see from the graph that $I$ is 40 for the lower line and 65 for the higher.

(c) *Suppose $\alpha = 0.25$. If this worker were to optimize using the two budget constraints you have identified with the two different $I$'s, how much leisure would he choose under each constraint? Can you illustrate what you find in a graph and tell from this where on the real AFDC budget constraint this worker will optimize?*

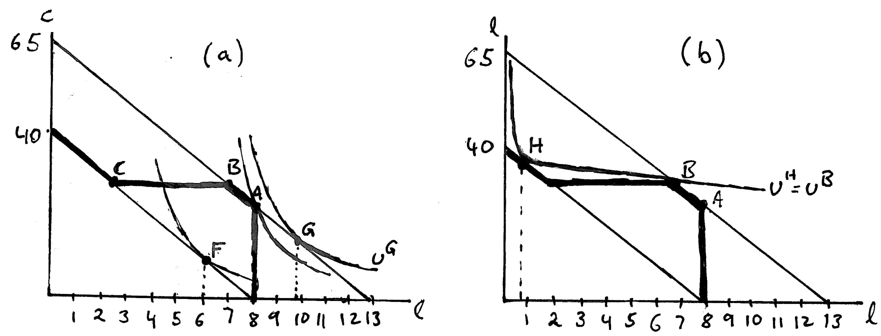
Answer: When $I = 40$, he would optimize at $\ell = (1-0.25)40/5 = 6$ and when $I = 65$, he would optimize at $\ell = (1-0.25)65/5 = 9.75$. This is illustrated in panel (a) of Graph 6.8 (on the next page) where $F$ with 6 hours of leisure occurs on the lower budget line and $G$ with 9.75 hours of leisure occurs on the higher. $F$ cannot be optimal inside the (bold) AFDC budget because it lies inside that budget. $G$, on the other hand, lies outside the (bold) AFDC budget and is therefore not feasible. But we do see that the indifference curve $u^G$ is steeper than -5 on the ray connecting the origin to the kink point $A$ — which implies the highest possible indifference curve on the bold AFDC budget goes through that kink point. Utility at $A = (8,25)$, for instance, would be $u(8,25) = 8^{0.75}25^{0.25} = 10.63$ while utility at $B = (7,30)$ is $u(7,30) = 7^{0.75}30^{0.25} = 10.07$. Thus, the real optimum when $\alpha = 0.25$ is bundle $A$ with no work and all leisure.

(d) *As $\alpha$ increases, what happens to the $MRS$ at each bundle?*

Answer: The $MRS$ for $u(\ell,c) = \ell^{1-\alpha} c^{\alpha}$ is

$$MRS = -\frac{\partial u/\partial \ell}{\partial u/\partial c} = -\frac{-(1-\alpha)\ell^{-\alpha} c^{\alpha}}{\alpha \ell^{1-\alpha} c^{\alpha-1}} = -\frac{(1-\alpha)c}{\alpha \ell}. \tag{6.27}$$

Thus, at any bundle $(\ell, c)$, the $MRS$ becomes larger in absolute value as $\alpha$ decreases and smaller in absolute value as $\alpha$ increases. Put differently, the slope of an indifference curve at any bundle becomes steeper as $\alpha$ gets smaller and shallower as $\alpha$ gets larger.




Graph 6.8: AFDC and Work Disincentives: Part 2

(e) *Repeat B(c) for* $\alpha = 0.3846$ *and for* $\alpha = 0.4615$. *What can you now say about this worker's choice for any* $0 < \alpha < 0.3846$*? What can you say about this worker's leisure choice if* $0.3846 < \alpha < 0.4615$*?*

Answer: When $\alpha = 0.3846$, $\ell = (1-0.3846)40/5 = 4.92$ at the lower budget line and $\ell = (1-0.3846)65/5 = 8$ on the higher budget line. The solution on the lower budget line lies inside the AFDC budget and is therefore not optimal. The solution of 8 hours of leisure on the higher budget, on the other hand, is within the AFDC budget — it is bundle *A*. Thus, when $\alpha = 0.3846$, the highest possible indifference curve on the AFDC budget is just tangent to the extended budget line $c = 65 - 5\ell$ at *A*. Since lower $\alpha$'s mean steeper indifference curves at every point, we can conclude from that that *A* will be optimal for all $\alpha$'s that lie between 0 and 0.3846. When $\alpha = 0.4615$, $\ell = (1-0.4615)40/5 = 4.31$ at the lower budget line and $\ell = (1-0.4615)65/5 = 7$ on the higher budget line. The solution on the lower budget is again inside the AFDC budget — so it cannot be optimal. The solution of 7 leisure hours on the higher budget, on the other hand, corresponds to *B* on the AFDC budget. Thus, when $\alpha = 0.4615$, the highest indifference curve on the AFDC budget is just tangent to the extended budget line $c = 65-5\ell$ at *B*. Since the slope of indifference curves becomes steeper as $\alpha$ falls, this implies that, for $\alpha$ between 0.3846 and 0.4615, the optimal leisure choice will lie in between *A* and *B* on the AFDC budget at $\ell = (1-\alpha)65/5 = 13(1-\alpha)$.

(f) *Repeat B(c) for* $\alpha = 0.9214$ *and calculate the utility associated with the resulting choice. Compare this to the utility of consuming at the kink point (7,30) and illustrate what you have found on a graph. What can you conclude about this worker's choice if* $0.4615 < \alpha < 0.9214$*?*

Answer: When $\alpha = 0.9214$, $\ell = (1-0.9214)40/5 = 0.629$ giving consumption of $w(8-\ell) = 5(8-0.629) = 36.856$. (On the higher budget line, $\ell = (1-0.9214)40/5 = 1.02$ which lies outside the AFDC budget). The bundle on the lower $c = 40-5\ell$ line, (0.629,36.856), gives utility $u(0.629, 36.856) = 0.629^{(1-0.9214)}36.856^{0.9214} = 26.76$. At *B*, the consumer would get utility $u(7,30) = 7^{(1-0.9214)}30^{0.9214} = 26.76$. Thus, the optimal bundle *H* on the budget line $c = 40-5\ell$ lies on the same indifference curve as *B* — as depicted in panel (b) of Graph 6.8. For $\alpha < 0.9214$, the indifference curve at *H* would be steeper and would therefore cut the AFDC budget while passing below *B* — and thus *B* is optimal for $\alpha$ just below 0.9214. Thus *B* is the optimal bundle for $0.4615 < \alpha < 0.9214$.

(g) *How much leisure will the worker take if* $0.9214 < \alpha < 1$*?*

Answer: Given that indifference curves become shallower at every bundle as $\alpha$ increases, we know that the indifference curve at *H* will be shallower for $\alpha > 0.9214$ than the one depicted in panel (b) of Graph 6.8. This implies that the optimal bundle for $\alpha > 0.9214$ lies to the left of *H* at $\ell = (1-\alpha)40/5 = 8(1-\alpha)$.

(h) *Describe in words what this tells you about what it would take for a worker to overcome the work disincentives under the AFDC program.*

Answer: The exponent $\alpha$ tells us how much weight a person places in his tastes on consumption rather than leisure. When $\alpha$ is high, consumption is valued much more than leisure — so even a small increase in consumption can justify giving up a lot of leisure. Thus, for very high $\alpha$, it is possible that someone with the AFDC budget constraint will in fact work close to full time despite the work disincentives. But that person's tastes would have to be pretty extreme — he would have to place virtually no value on leisure time. For anyone that places some non-trivial value on leisure time — which implies $\alpha$ isn't close to 1 or, to be more precise, $\alpha < 0.9214$ — the payoff from working close to full time is simply not high enough to sacrifice that much leisure. Thus, for most values of $\alpha$, the person will choose to work less than 1 hour per day.

# Conclusion: Potentially Helpful Reminders

1. Remember what we mean by "at the margin." It is a concept we have been developing throughout the chapters leading to this one, and we are now beginning to use the phrase more frequently. When we use the phrase, we mean that something holds "around the bundle" on which we are focused. For instance, to say that all consumers who face the same prices and who optimize at an interior solution have the same tastes "at the margin" is the same as saying that such consumers value the goods the same around the bundle that they have chosen as their best bundle.

2. If within-chapter-exercise 6.4 makes sense to you, you have come a long way toward understanding why markets in which everyone is charged the same price result in efficient outcomes where no further mutually beneficial trades are possible. End-of-chapter exercise 6.1 is also useful in this regard.

3. Remember that "efficient" means "no one can be made better off without someone else being made worse off." And "inefficient" means that someone can be made better off without anyone being made worse off. Economists don't like inefficiency because something is being left on the table, so to speak.

4. Some of what we will cover in later chapters will become a lot easier if you can take a minute to get comfortable with the technical definition of a "convex set". A set of points is convex if the line connecting any two points in the set is also fully contained within the set; otherwise it is non-convex. A filled in circle is a convex set. A donut is not.

5. End-of-chapter exercise 6.14 is a good lead-in to Chapter 7 which starts with a similar example.