CHAPTER

7

Income and Substitution Effects in Consumer Goods Markest

In Chapter 6 we showed how economic circumstances combine with tastes to result in choice or behavior. In Chapter 7 we show how consumer choices (and thus the consumer behavior we observe) change as circumstances change — i.e. as incomes and prices change. Put differently, we will now show how "people respond to incentives" in the consumer goods market.

Chapter Highlights

The main points of the chapter are:

1. There are **two ways in which economic circumstances typically change**: a change in income and a change in opportunity costs.

2. When only **income changes**, we can predict the change in behavior if we know something about **how indifference curves relate to one another** — because we jump from one indifference curve to another. Whether tastes are quasilinear or homothetic, whether goods are normal or inferior — these are statements about that relationship between indifference curves.

3. When only **opportunity costs change** and *real* income remains constant, we don't need to know anything about the relationship of indifference curves to one another — because the change in behavior occurs along a single indifference curve. Thus, **the *shape* of the relevant indifference curve is all that matters** — which is the same as saying that the degree of substitutability of the goods at the margin is all that matters.

4. **Substitution effects** arise as we slide along indifference curves because opportunity costs have changed; **income effects** arise as we jump between indifference curves because real income has changed.

5. **Price changes give rise to both of these effects**. To identify the substitution effect, we only look at the initial indifference curve and thus need to know about the substitutability of goods at the margin; to identify income effects, we have to know how indifference curves relate to one another.

6. In the calculus-based material of Part B of *Microeconomics: An Intuitive Approach with Calculus*, we show how **constrained utility maximization** gives us the choices that people make as incentives change while the **constrained expenditure minimization** problem allows us to disentangle the substitution effect from the income effect.

Using the LiveGraphs

For an overview of what is contained on the LiveGraphs site for each of the chapters (from Chapter 2 through 29) and how you might utilize this resource, see pages 2-3 of Chapter 1 of this *Study Guide*. To access the LiveGraphs for Chapter 7, click the *Chapter 7* tab on the left side of the LiveGraphs web site.

In addition to the *Animated Graphics*, *Static Graphics* and *Downloads* portions of the LiveGraphs site, this Chapter has several **Exploring Relationships** modules:

1. The module titled "The Income and Substitution Effect for Two Goods" allows you to review all possible 2-good gases for income and substitution effects.

2. The two modules labeled *Interactive Application* allow you to practice with income effects (in the application entitled "Normal/Inferior Goods – Income Change") and with both income and substitution effects (in the application entitled "The Income and Substitution Effect Game").

7A Solutions to Within-Chapter-Exercises for Part A

Exercise 7A.1 *Is it also the case that whenever there is a positive income effect on our consumption of one good, there must be a negative income effect on our consumption of a different good?*

Answer: No — since it is possible for our consumption of all goods to go up as income increases, the income effect could be positive for all goods.

Exercise 7A.2 *Can a good be an inferior good at all income levels? (*Hint*: Consider the bundle (0,0).)*

Answer: No. The reason is that, in order for a good to be inferior, it must be that you consume more of it as income falls. But, as income falls toward zero, at some point it will not be possible to consume more as income falls — because there simply won't be enough income to consume more. Thus, around the origin, no good can be inferior.

Exercise 7A.3 *Are all inferior goods necessities? Are all necessities inferior goods? (*Hint*: The answer to the first is yes; the answer to the second is no.) Explain.*

Answer: If you consume less of a good as income goes up, then it must be true that you spend a smaller fraction of your income on that good as income goes up. Thus, all inferior goods are necessities. At the same time, it may be the case that the fraction of your income spent on a good declines as your income goes up — but you still buy more of the good. (For instance, suppose your income goes up by 10% and you choose to consume 5% more of a good. Then the fraction of income spent on that good is declining even though you are increasing your consumption of the good as your income goes up.) Thus, necessities could be normal goods.

Exercise 7A.4 *At a particular consumption bundle, can both goods (in a 2-good model) be luxuries? Can they both be necessities?*

Answer: No. In a 2-good model, you will end up spending all your income as income increases. So suppose you are currently spending all your income on the two goods and your income now increases by 10%. If your consumption of both goods increases by more than 10%, then you would now be spending more than your new income. If your consumption of both goods increases by less than 10%, you would be spending less than your new income.

Exercise 7A.5 *If you knew only that my brother and I had the same income (but not necessarily the same tastes), could you tell which one of us drove more miles — the one that rented or the one that took taxis?*

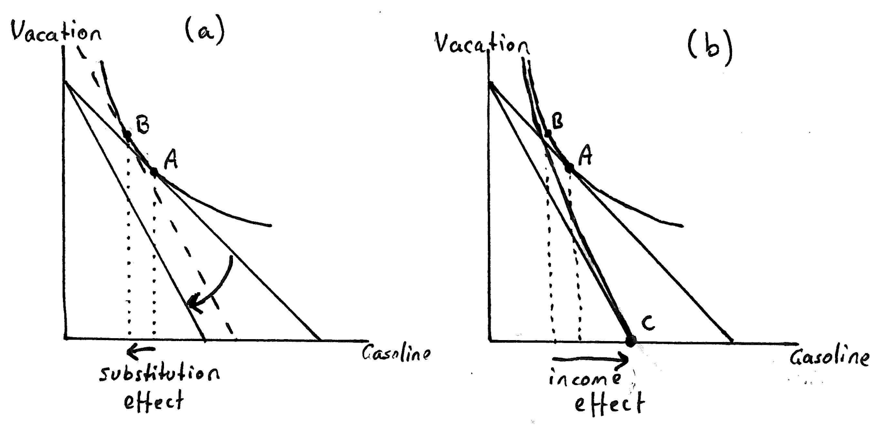
Answer: Yes. Suppose my brother faces the intersecting budget lines — with the steeper one representing taxis and the shallower one representing rental cars. He chooses the steeper (taxi) budget line. Then we know that he must be consuming a bundle to the left of the intersection point of the two lines — because if he chose to the right of that point, he could have had more of everything on the shallower budget and thus should have chosen the shallower (rental car) budget instead. Thus, by choosing the steeper taxi budget, we know my brother consumes to the left of the intersection point. I, on the other hand, chose the shallower rental car budget. If I were to then choose a bundle to the left of the intersection point, I could have done better choosing the steeper budget because I could get more of both goods. Thus I must be consuming to the right of the intersection point. If my brother and I have the same incomes (and thus face the same taxi and rental car budgets), it therefore must be the case that my brother consumes to the left of the intersection point on the taxi budget and I consume to the right on the rental car budget. We can unambiguously say I consumed more miles driven.

Exercise 7A.6 True or False*: If you observed my brother and me consuming the same number of miles driven during our vacations, then our tastes must be those of perfect complements between miles driven and other consumption.*

Answer: It would at a minimum have to be the case that the indifference curve at the intersection of the two budget lines has a sharp kink at that point. That kink could be such that it forms a right angle — thus creating the typical perfect complements indifference curve. But at a minimum it has to be such that the upper part of the indifference curve is steeper than the taxi budget and the lower part is shallower than the rental car budget — with a kink at the intersection.

Exercise 7A.7 *Can you re-tell the Heating Gasoline-in-Midwest story in terms of income and substitution effects in a graph with "yearly gallons of gasoline consumption" on the horizontal axis and "yearly time on vacation in Florida" on the vertical?*

Answer: In panel (a) of Graph 7.1, bundle A is the original consumption bundle prior to the increase in the price of gasoline. The increase in the price of gasoline then rotates the budget clockwise. Bundle B lies on the compensated budget at the new price of gasoline — and the move from A to B is the substitution effect. As always, the substitution effect causes a decrease in consumption of the good (gasoline) that has become more expensive.



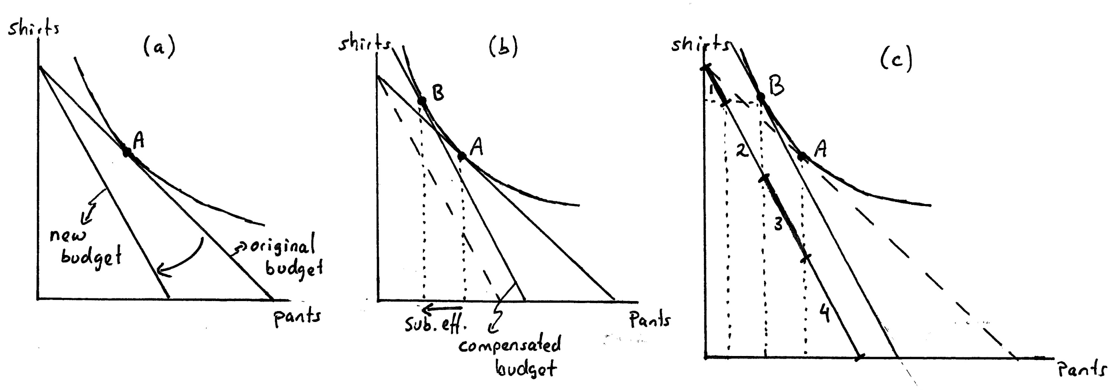
Graph 7.1: Gasoline and Florida Vacation Time

Panel (b) illustrates C — with no consumption of Florida vacation time. This corner solution is rationalized by an indifference curve that crosses the new budget at C — creating an income effect in the opposite direction of the substitution effect.

Since the income effect is larger than the substitution effect, the consumer shifts from *A* before the increase in the price of gasoline to *C* after the price increase — with an overall increase in gasoline consumption resulting from the price increase.

Exercise 7A.8 *Replicate Graph 7.7 for an increase in the price of pants (rather than a decrease).*

Answer: Panel (a) of Graph 7.2 illustrates the original consumption bundle *A* and the change in the budget constraint when the price of pants increases. Panel (b) illustrates the compensated budget and the resulting bundle *B* — with the substitution effect as the movement from *A* to *B*. As always, this effect says the consumer will consume less of what has become more expensive, more of what has become relatively cheaper. Finally, panel (c) identifies four regions (labeled 1, 2, 3 and 4) on the new (uncompensated) budget line. If the consumer ends up optimizing in region 1, her consumption of pants decreases and her consumption of shirts increases with a decline in income (relative to the compensated budget) — which implies that pants are a normal good and shirts are inferior. In region 2, the consumption of both goods declines with income — thus both pants and shirts are normal goods. In regions 3 and 4, consumption of shirts decreases and consumption of pants increases with a drop in income (from the compensated budget) — thus making shirts normal and pants inferior. In region 3, however, the consumer still buys fewer pants as the price increases (i.e. *C* is to the left of *A*) — which means pants are regular inferior; in region 4, on the other hand, pants consumption goes up with an increase in price, which makes pants a Giffen good.



Graph 7.2: Gasoline and Florida Vacation Time

7B Solutions to Within-Chapter-Exercises for Part B

Exercise 7B.1 *Set up my brother's constrained optimization problem and solve it to check that his optimal consumption bundle is indeed equal to this.*

Answer: My brother's optimization problem is

$$\max_{x_1,x_2} u(x_1,x_2) = x_1^{0.1}x_2^{0.9} \text{ subject to } x_1 + x_2 = 2000, \tag{7.1}$$

which gives rise to the Lagrange function

$$\mathscr{L}(x_1,x_2,\lambda) = x_1^{0.1}x_2^{0.9} + \lambda(2000 - x_1 - x_2). \tag{7.2}$$

The first two first order conditions for this problem are then

$$\begin{aligned} \frac{\partial \mathscr{L}}{\partial x_1} &= 0.1x_1^{-0.9}x_2^{0.9} - \lambda = 0, \\ \frac{\partial \mathscr{L}}{\partial x_2} &= 0.9x_1^{0.1}x_2^{-0.1} - \lambda = 0. \end{aligned} \tag{7.3}$$

Moving λ to the other side of each equation, dividing the equations by one another and solving for x_1 gives us $x_1 = x_2/9$. Substituting this into the budget constraint $x_1 + x_2 = 2000$, we get $x_2/9 + x_2 = 2000$ which solves to $x_2 = 1,800$. Plugging this back into $x_1 = x_2/9$ furthermore gives $x_1 = 200$.

Exercise 7B.2 *How much did I pay in a fixed rental car fee in order for me to be indifferent in this example to taking taxis? Why is this amount larger than in the Cobb-Douglas case we calculated earlier?*

Answer: At B, I am consuming 2,551 miles at a per-mile cost of \$0.2 — for a total of \$510.20. At that bundle, I am also consuming approximately \$918 in other consumption. Thus, I am spending a total of approximately $\$918 + \$510 = \$1,428$ after having paid the fixed fee for the rental car. Since I started with \$2,000, that means the rental car fee must have been $\$2000 - \$1428 = \$572$. This amount is larger than under Cobb-Douglas preferences because the implicit elasticity of substitution is now 2 rather than 1.

Exercise 7B.3 *Check to see that this solution is correct.*

Answer: The Lagrange function for this optimization problem is

$$\mathscr{L}(x_1,x_2,\lambda) = x_1^{0.5}x_2^{0.5} + \lambda(200 - p_1x_1 - 10x_2). \tag{7.4}$$

The first two first order conditions for this problem are then

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial x_1} &= 0.5x_1^{-0.5}x_2^{0.5} - \lambda p_1 = 0,\\ \frac{\partial \mathscr{L}}{\partial x_2} &= 0.5x_1^{0.5}x_2^{-0.5} - 10\lambda = 0.\end{aligned} \tag{7.5}$$

Moving the λ terms to the other side, dividing the equations by one another and then solving for x_1, we get $x_1 = 10x_2/p_1$. Plugging this into the budget constraint $p_1x_1 + 10x_2 = 200$ and solving for x_2, we get $x_2 = 10$, and plugging this back into $x_1 = 10x_2/p_1$, we get $x_1 = 100/p_1$.

Exercise 7B.4 *Verify the above solutions to the minimization problem.*

Answer: The Lagrange function for this optimization problem is

$$\mathscr{L}(x_1, x_2, \lambda) = 10x_1 + 10x_2 + \lambda(u^A - x_1^{0.5}x_2^{0.5}). \tag{7.6}$$

The first two first order conditions for this problem are then

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial x_1} &= 10 - 0.5\lambda x_1^{-0.5}x_2^{0.5} = 0,\\ \frac{\partial \mathscr{L}}{\partial x_2} &= 10 - 0.5\lambda x_1^{0.5}x_2^{-0.5} = 0.\end{aligned} \tag{7.7}$$

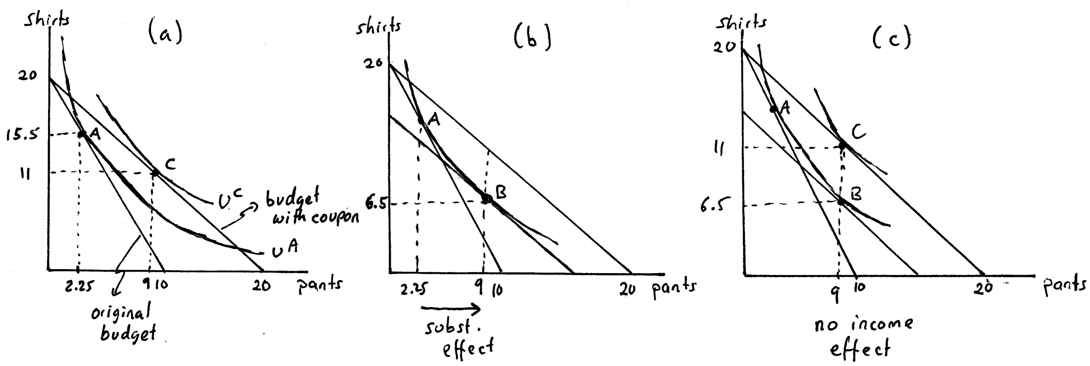
Solving these for x_1 in the usual way gives us $x_1 = x_2$. Plugging this into the constraint $u^A = x_1^{0.5}x_2^{0.5}$, we then get $x_2 = u^A$, and — given we concluded $x_1 = x_2$, $x_1 = u^A$. Since $u^A \approx 7.071$, this implies $x_1 = x_2 \approx 7.071$.

Exercise 7B.5 *Notice that the ratio of my pants to shirts consumption is the same* $(= 1)$ *at bundles B and C. What feature of Cobb-Douglas tastes is responsible for this result?*

Answer: Cobb-Douglas tastes are homothetic — which implies that optimal consumption bundles lie on the same ray from the origin for all income levels (assuming no price changes).

Exercise 7B.6 *Using the calculations above, plot graphs similar to Graph 7.10 illustrating income and substitution effects when my tastes can be represented by the utility function* $u(x_1, x_2) = 6x_1^{0.5} + x_2$.

Answer: This is done in Graph 7.3 (on the next page). Notice again that there is no income effect relative to the good x_1 (pants) — which is because of the fact that the utility function represents tastes that are quasilinear in x_1. (Quasilinear goods have no income effects.)



Graph 7.3: Pants and Shirts with Quasilinear Tastes

End of Chapter Exercises

Exercise 7.5

Return to the analysis of my undying love for my wife expressed through weekly purchases of roses (as introduced in end-of-chapter exercise 6.4).

A: *Recall that initially roses cost $5 each and, with an income of $125 per week, I bought 25 roses each week. Then, when my income increased to $500 per week, I continued to buy 25 roses per week (at the same price).*

(a) *From what you observed thus far, are roses a normal or an inferior good for me? Are they a luxury or a necessity?*

Answer: As income went up, my consumption remained unchanged. This would typically indicate that the good in question is borderline normal/inferior — or quasilinear. Since the consumption at the lower income is at a corner solution, however, we cannot be certain that the good is not inferior, with the MRS at the original optimum larger in absolute value than the MRS at the new (higher income) optimum. Regardless, roses must be a necessity — whether they are borderline inferior/normal or inferior, the percentage of income spent on roses declines as income increases.

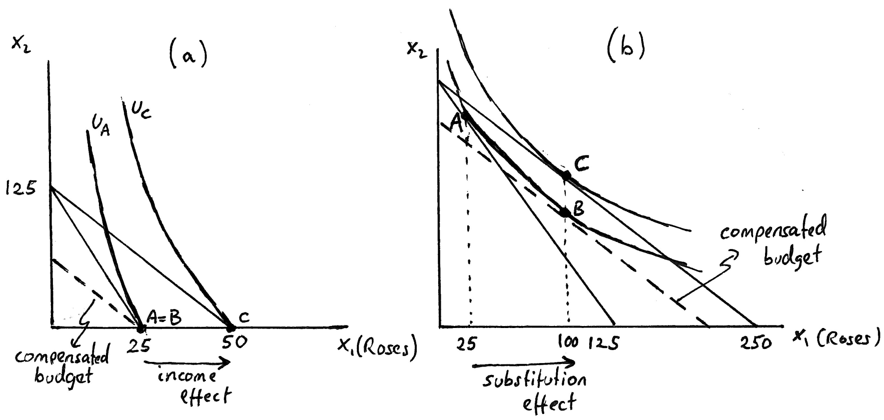
(b) *On a graph with weekly roses consumption on the horizontal and "other goods" on the vertical, illustrate my budget constraint when my weekly income is $125. Then illustrate the change in the budget constraint when income remains $125 per week and the price of roses falls to $2.50. Suppose that my optimal consumption of roses after this price change rises to 50 roses per week and illustrate this as bundle C.*

Answer: This is illustrated in panel (a) of Graph 7.4 (on the next page) where A is the original corner solution, C is the new corner solution and the dashed line is the compensated budget.

(c) *Illustrate the compensated budget line and use it to illustrate the income and substitution effects.*

Answer: This is also illustrated in panel (a) of the graph. In this case, there is no substitution effect (in terms of roses) and only an income effect.

(d) *Now consider the case where my income is $500 and, when the price changes from $5 to $2.50, I end up consuming 100 roses per week (rather than 25). Assuming quasilinearity in roses, illustrate income and substitution effects.*



Graph 7.4: Love and Roses

Answer: This is illustrated in panel (b) of Graph 7.4 where the dashed line is again the compensated budget line. Unlike in panel (a), the entire change in roses consumption is now due to a substitution effect rather than an income effect.

(e) True or False*: Price changes of goods that are quasilinear give rise to no income effects for the quasilinear good unless corner solutions are involved.*

Answer: This is true. We will often make the statement that income effects disappear if we assume quasilinearity of a good — because then a good is borderline normal/inferior, which implies consumption remains unchanged as income changes. This is true so long as the consumer is at an interior solution. If quasilinear tastes lead to corner solutions, then this may give rise to income effects as we see in panel (a) of the graph.

B: *Suppose again, as in 6.4B, that my tastes for roses (x_1) and other goods (x_2) can be represented by the utility function $u(x_1,x_2) = \beta x_1^{\alpha} + x_2$.*

(a) *If you have not already done so, assume that p_2 is by definition equal to 1, let $\alpha = 0.5$ and $\beta = 50$, and calculate my optimal consumption of roses and other goods as a function of p_1 and I.*

Answer: Solving the optimization problem

$$\max_{x_1,x_2} 50x_1^{0.5} + x_2 \text{ subject to } I = p_1x_1 + x_2, \tag{7.8}$$

we get

$$x_1 = \frac{625}{p_1^2} \text{ and } x_2 = I - \frac{625}{p_1}. \tag{7.9}$$

(b) *The original scenario you graphed in 7.5A(b) contains corner solutions when my income is \$125 and the price is initially \$5 and then \$2.50. Does your answer above allow for this?*

Answer: Substituting $I = 125$ and $p_1 = 5$ into our equations (7.9) for x_1 and x_2 from above, we get $x_1 = 625/(5^2) = 25$ and $x_2 = 125 - (625/5) = 0$. This is exactly the original corner solution in the scenario in part A.

Changing the price to $p_1 = 2.5$, we get $x_1 = 625/(2.5^2) = 100$ and $x_2 = 125-(625/2.5) = -125$. Given that the solution from our Lagrange method now gives us a negative consumption level for x_2, we know that the true optimum is the corner solution where all income is spent on x_1 — i.e. the bundle (50,0) just as described in the scenario in A.

At the original price, it turns out that the MRS at the corner solution is exactly equal to the slope of the budget line. At the lower price, the MRS is large in absolute value than the budget line — which means the indifference curve cuts the budget line at the corner from above. The tangency of an indifference curve with this budget line therefore does not happen until x_2 is negative — which the Lagrange method finds but which is not economically meaningful.

(c) *Verify that the scenario in your answer to 7.5A(d) is also consistent with tastes described by this utility function — i.e. verify that A, B and C are as you described in your answer.*

Answer: Using equations (7.9), we get $x_1 = 625/(5^2) = 25$ and $x_2 = 500 - (625/5) = 375$ when $p_1 = 5$ (and $I = 500$), and we get $x_1 = 625/(2.5^2) = 100$ and $x_2 = 500 - (625/2.5) = 250$ when $p_1 = 2.5$. These correspond to A and C in panel (b) of Graph 7.4.

To calculate B in the graph, we need to first find the utility level associated with the original bundle A — i.e. $u(25,375) = 50(25^{0.5}) + 375 = 625$. We then need to find what bundle the consumer would buy if she was given enough money to reach that same indifference curve at the new price; i.e. we need to solve the problem

$$\min_{x_1,x_2} 2.5x_1 + x_2 \text{ subject to } 625 = 50x_1^{0.5} + x_2. \tag{7.10}$$

Solving the first order conditions, we then get $x_1 = 100$ and $x_2 = 125$ — consistent with panel (b) of the graph.

Exercise 7.8: *Sam's Club and the Marginal Consumer*

Business Application: *Sam's Club and the Marginal Consumer: Superstores like Costco and Sam's Club serve as wholesalers to businesses but also target consumers who are willing to pay a fixed fee in order to get access to the lower wholesale prices offered in these stores. For purposes of this exercise, suppose that you can denote goods sold at Superstores as x_1 and "dollars of other consumption" as x_2.*

A: *Suppose all consumers have the same homothetic tastes over x_1 and x_2 but they differ in their income. Every consumer is offered the same option of either shopping at stores with somewhat higher prices for x_1 or paying the fixed fee c to shop at a Superstore at somewhat lower prices for x_1.*

(a) *On a graph with x_1 on the horizontal axis and x_2 on the vertical, illustrate the regular budget (without a Superstore membership) and the Superstore budget for a consumer whose income is such that these two budgets cross on the 45 degree line. Indicate on your graph a vertical distance that is equal to the Superstore membership fee c.*

Answer: In panel (a) of Graph 7.5 (on the next page), the Superstore budget has shallower slope (because of the lower price of x_1) but a lower vertical intercept (because of the fixed membership fee). The lower two budgets in the graph are such that they intersect on the 45 degree line.

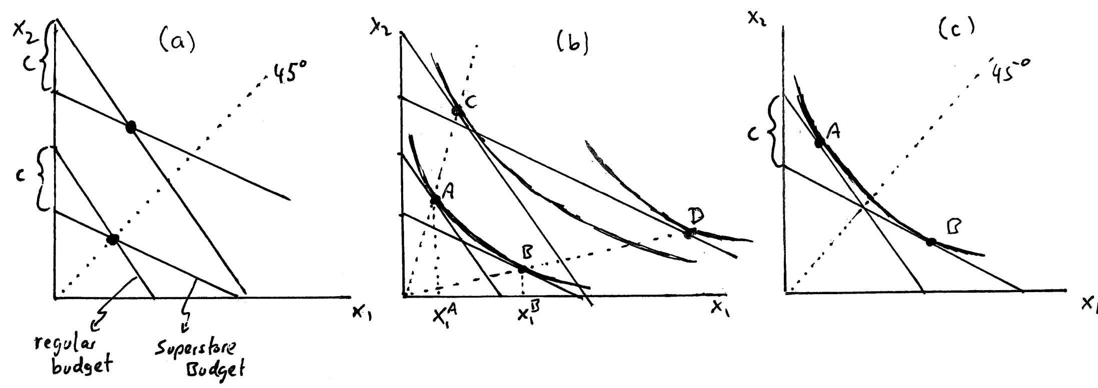
(b) *Now consider a consumer with twice that much income. Where will this consumer's two budgets intersect relative to the 45 degree line?*

Answer: This is also illustrated in panel (a). When income is doubled, the vertical intercept of the regular budget doubles — but the vertical intercept of the Superstore budget more than doubles because the fixed fee remains the same. (If the initial income is I, the initial intercept of the Superstore budget is $(I - c)$. When income doubles, the new intercept is $(2I - c)$ — which is greater than $2(I - c)$.) For this reason, the two budget lines will cross above the 45 degree line when income doubles.

(c) *Suppose consumer 1 (from part (a)) is just indifferent between buying and not buying the Superstore membership. How will her behavior differ depending on whether or not she buys the membership.*

Answer: In panel (b) of the graph, Consumer 1 will then consume at bundle A if she does not buy the membership and at bundle B if she does. This is a pure substitution effect — with greater consumption when price is lower.

(d) *If consumer 1 was indifferent between buying and not buying the Superstore membership, can you tell whether consumer 2 (from part (b)) is also indifferent? (*Hint*: Given that tastes*



Graph 7.5: Sam's Club

are homothetic and identical across consumers, what would have to be true about the intersection of the two budgets for the higher income consumer in order for the consumer to also be indifferent between them?)

Answer: Consumer 2 will then definitely buy the membership. This is also illustrated in panel (b) of Graph 7.5 where C is the optimal bundle on the regular budget and D is the optimal bundle on the Superstore budget for the higher income consumer. (These optimal bundles lie along rays from the origin going through A and B because we are assuming that tastes are homothetic). Because of the different relationship between the two budgets for the lower and higher income consumers (as identified in panel (a)), D lies on a higher indifference curve than C — implying that consumer 2 will buy the membership.

(e) True or False*: Assuming consumers have the same homothetic tastes, there exists a "marginal" consumer with income* $\overline{I}$ *such that all consumers with income greater than* $\overline{I}$ *will buy the Superstore membership and no consumer with income below* $\overline{I}$ *will buy that membership.*

Answer: This is true. Higher income consumers whose two budgets will intersect above the 45 degree line will be better off on the Superstore budget (as illustrated in panel (b)). For analogous reasons, lower income consumers will face that intersection point below the 45 degree line — causing the regular budget to yield an optimum with greater utility than the Superstore budget.

(f) True or False*: By raising c and/or* p_1*, the Superstore will lose relatively lower income customers and keep high income customers.*

Answer: True. Suppose we begin again with Consumer 1 who is indifferent and whose budget lines are illustrated again in panel (c) of Graph 7.5. An increase in c will cause the shallower Superstore budget to shift in parallel — causing the two budgets to intersect below the 45 degree line and leaving Consumer 1 better off on the regular budget (where she can still consume at A). If p_1 increases in the Superstore, the slope of the Superstore budget becomes steeper — again causing the intersection point to fall below the 45 degree line and leaving Consumer 1 better off at A under the regular budget. Thus, the marginal consumer will cease shopping at the Superstore if c or p_1 are increased. Because of the homotheticity assumption, we also know that the new marginal consumer will again have her budgets intersect on the 45 degree line — and we have seen in panel (a) that this intersection point moves up on the regular budget as income increases. If an increase in c or p_1 have caused the intersection point to slide below the 45 degree line for the original marginal consumer, then an increase in income will cause it to slide back up. Thus, there exists some higher income level at which we will find our new marginal consumer.

(g) *Suppose you are a Superstore manager and you think your store is overcrowded. You'd like to reduce the number of customers while at the same time increasing the amount each customer purchases. How would you do this?*

Answer: You would want to increase c — which will raise the income of your marginal consumer and reduce the overall number of consumers with memberships. Then, in order to get your members to shop more, you would lower p_1 — but not so much that membership again goes up by too much. You can see that this is possible by again looking at panel (c) of the graph. By increasing c, you insure that this marginal consumer will no longer be a member. You can then lower price (which will make the new budget shallower) and keep the marginal consumer from coming back to your store so long as you don't lower the prices too much.

B: *Suppose you manage a Superstore and you are currently charging an annual membership fee of \$50. Since x_2 is denominated in dollar units, $p_2 = 1$. Suppose that $p_1 = 1$ for those shopping outside the Superstore but your store sells x_1 at 0.95. Your statisticians have estimated that your consumers have tastes that can be summarized by the utility function $u(x_1, x_2) = x_1^{0.15} x_2^{0.85}$.*

(a) *What is the annual discretionary income (that could be allocated to purchasing x_1 and x_2) of your "marginal" consumer?*

Answer: The marginal consumer is indifferent between buying and not buying the membership. If she does not buy the membership, her budget is $x_1 + x_2 = I$ — and she would optimize by solving

$$\max_{x_1, x_2} x_1^{0.15} x_2^{0.85} \text{ subject to } x_1 + x_2 = I. \tag{7.11}$$

This gives us $x_1 = 0.15I$ and $x_2 = 0.85I$. Thus, without membership, our consumer gets utility

$$u(0.15I, 0.85I) = (0.15I)^{0.15}(0.85I)^{0.85} \approx 0.655I. \tag{7.12}$$

If she becomes a member at the Superstore, her budget will be $0.95x_1 + x_2 = I - 50$. She will then solve the maximization problem

$$\max_{x_1, x_2} x_1^{0.15} x_2^{0.85} \text{ subject to } 0.95x_1 + x_2 = I - 50. \tag{7.13}$$

This gives us $x_1 = 0.15(I-50)/0.95 \approx 0.158(I-50)$ and $x_2 = 0.85(I-50)$. Plugging these into the utility function gives

$$u(0.158(I-50), 0.85(I-50)) = (0.158(I-50))^{0.15}(0.85(I-50))^{0.85} \approx 0.660(I-50). \tag{7.14}$$

In order for the consumer to be indifferent, it must then be that the utility under the regular budget equals the utility under the Superstore budget — i.e. $0.655I = 0.660(I-50)$. Solving for I, we get that the income of the marginal consumer is $I \approx \$6,600$. (Without rounding along the way, the figure is \$6,524.)

(b) *Can you show that consumers with more income than the marginal consumer will definitely purchase the membership while consumers with less income will not? (*Hint*: Calculate the income of the marginal consumer as a function of c and show what happens to income that makes a consumer marginal as c changes.)*

Answer: We can solve the same problems as in B(a) above but now let the membership fee be equal to c rather than \$50. The optimal bundle under the regular budget does not change; the optimal bundle under the Superstore budget is now $(0.158(I-c), 0.85(I-c))$, giving us utility under the Superstore constraint of approximately $0.66(I-c)$. Setting that equal to the utility under the regular budget $(0.655I)$ and solving for I we get

$$I \approx 132c. \tag{7.15}$$

Thus, as c increases, the income of the marginal consumer increases, and as c decreases, the income of the marginal consumer decreases.

(c) *If the membership fee is increased from \$50 to \$100, how much could the Superstore lower p_1 without increasing membership beyond what it was when the fee was \$50 and p_1 was 0.95?*

Answer: In order to keep membership constant, the marginal consumer who buys the membership before and after the changes must be the same. The optimal bundle under the regular budget would again be unchanged — i.e. $x_1 = 0.15I$ and $x_2 = 0.85I$, giving utility of $u \approx 0.655I$. Letting the new Superstore price be p_1, a consumer with a membership would solve

$$\max_{x_1,x_2} x_1^{0.15}x_2^{0.85} \text{ subject to } p_1x_1 + x_2 = I - 100. \quad (7.16)$$

This gives us $x_1 = 0.15(I-100)/p_1$ and $x_2 = 0.85(I-100)$. Plugging this into the utility function, we get that

$$u\left(\frac{0.15(I-100)}{p_1}, 0.85(I-100)\right) \approx \frac{0.655}{p_1^{0.15}}(I-100). \quad (7.17)$$

We calculated in B(a) that the marginal consumer when $p_1 = 0.95$ and $c = 50$ had income of approximately \$6,600. In order for this consumer to be indifferent once again after the membership fee goes up to \$100, it has to be that her utility without the membership is again equal to her utility with it; i.e.

$$0.655(6600) = \frac{0.655}{p_1^{0.15}}(6600-100). \quad (7.18)$$

Solving this for p_1, we get $p_1 \approx 0.903$.

Exercise 7.12: *Fuel Efficiency, Gasoline Consumption, and Gas Prices*

Policy Application: *Fuel Efficiency, Gasoline Consumption and Gas Prices: Policy makers frequently search for ways to reduce consumption of gasoline. One straightforward option is to tax gasoline — thereby encouraging consumers to drive less in their cars and switch to more fuel efficient cars.*

A: *Suppose that you have tastes for driving and for other consumption, and assume throughout that your tastes are homothetic.*

(a) *On a graph with monthly miles driven on the horizontal and "monthly other consumption" on the vertical axis, illustrate two budget lines: One in which you own a gas-guzzling car — which has a low monthly payment (that has to be made regardless of how much the car is driven) but high gasoline use per mile; the other in which you own a fuel efficient car — which has a high monthly payment that has to be made regardless of how much the car is driven but uses less gasoline per mile. Draw this in such a way that it is possible for you to be indifferent between owning the gas-guzzling and the fuel-efficient car.*

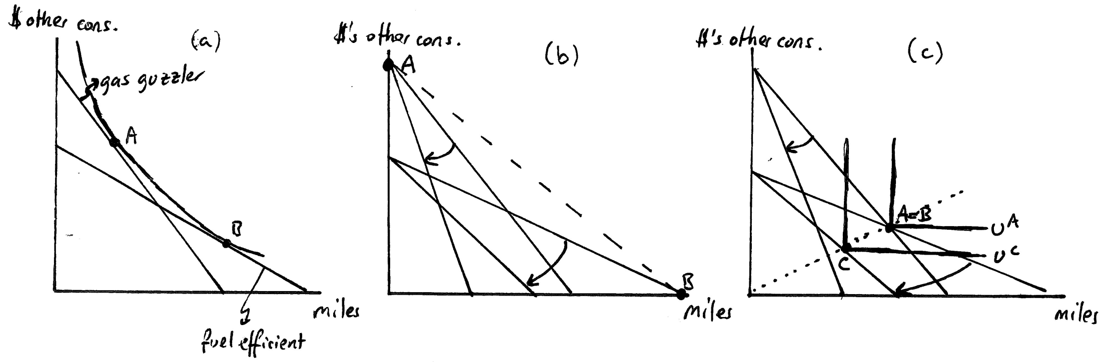
Answer: These are illustrated in panel (a) of Graph 7.6 (on the next page) where the gas guzzler budget has a steeper slope (because of the higher opportunity cost of driving from the greater gasoline use) and higher intercept (because of the lower monthly payments for the car.)

(b) *Suppose you are indeed indifferent. With which car will you drive more?*

Answer: Your optimal consumption bundle is A under the gas guzzler budget and B under the fuel efficient budget — and because of the implicit substitution effect, you will drive more in the fuel efficient car.

(c) *Can you tell with which car you will use more gasoline? What does your answer depend on?*

Answer: No, you cannot tell. In the fuel efficient car, you will use less gasoline per mile but you will also drive more miles. Whether or not you use more gasoline in the fuel efficient car or the gas guzzler depends on which effect dominates. This depends on how far apart A and B are — i.e. on how large the substitution effect is. If driving and other consumption are sufficiently substitutable, then you will use more gasoline when you drive the fuel efficient



Graph 7.6: Cars, Gasoline Taxes and Fuel Efficiency

car; if, on the other hand, driving and other goods are sufficiently complementary, you will use less gasoline in the fuel efficient car.

(d) *Now suppose that the government imposes a tax on gasoline — and this doubles the opportunity cost of driving both types of cars. If you were indifferent before the tax was imposed, can you now say whether you will definitively buy one car or the other (assuming you waited to buy a car until after the tax is imposed)? What does your answer depend on? (*Hint*: It may be helpful to consider the extreme cases of perfect substitutes and perfect complements before deriving your general conclusion to this question.)*

Answer: As suggested in the hint, the answer is easiest to see if you start by looking at extremes. In panel (b) of the graph, we assume that driving and other goods are perfect substitutes. In order for someone to be indifferent between the two budgets, the optimal bundles must lie on opposite corners of the two budgets. (The indifference curve is illustrated as the dashed line connecting A and B). If the price of gasoline now increases due to the tax, A remains possible but B does not — which means that now the person would choose A. In other words, conditional on buying one of the two types of cars, the person would choose the gas guzzler. The example is a bit artificial in the sense that someone who will not end up driving at all would presumably not buy a car at all — but you can see how the logic also holds for tastes that are close to perfect substitutes where the consumer would choose interior solutions.

In panel (c) of the graph, we go to the other extreme — with tastes over miles and other consumption modeled as perfect complements. In that case, A=B to begin with — and when gasoline prices go up, the fuel efficient car becomes unambiguously better (with the optimum at C and all bundles on the after-tax gas guzzling budget falling below u^C.)

Upon reflection, this should make intuitive sense. If miles and other consumption are relatively complementary, then it makes sense to switch to a more fuel efficient car because we want to keep driving quite a bit even if the price of gasoline increases. If, on the other hand, miles and other consumption are relatively substitutable, then one way to respond to a price increase is to substitute away from gasoline altogether and just drive very little. With only a little driving each month, it's better to pay the lower fixed cost of the gas guzzler even if each mile costs more.

(e) *The empirical evidence suggests that consumers shift toward more fuel efficient cars when the price of gasoline increases.* True or False*: This would tend to suggest that driving and other good consumption are relatively complementary.*

Answer: True — based on the explanation to part (d) above.

(f) *Suppose an increase in gasoline taxes raises the opportunity cost of driving a mile with a fuel efficient car to the opportunity cost of driving a gas guzzler* before *the tax increase. Would someone with homothetic tastes drive more or less in the fuel efficient car after the tax increase than she would in a gas guzzler prior to the tax increase?*

Answer: The person would drive less in the fuel efficient car after the tax increase than in the gas guzzler before the tax increase. You can illustrate this simply by drawing the gas guzzler budget before the tax and the fuel efficient budget after the tax. You should get both budgets to have the same slope (because of the same opportunity cost of driving) — but the fuel efficient car has lower intercept because of the higher monthly payments. This is then a pure income effect — with the new optimal bundle on the after-tax fuel efficient budget lying on the same ray from the origin as the original optimal bundle before-tax gas guzzler budget. The new bundle then necessarily lies to the left of the original.

B: *Suppose your tastes were captured by the utility function $u(x_1,x_2) = x_1^{0.5}x_2^{0.5}$, where x_1 stands for miles driven and x_2 stands for other consumption. Suppose you have $600 per month of discretionary income to devote to your transportation and other consumption needs and that the monthly payment on a gas-guzzler is $200. Furthermore, suppose the initial price of gasoline is $0.10 per mile in the fuel efficient car and $0.20 per mile in the gas-guzzler.*

(a) *Calculate the number of monthly miles driven if you own a gas-guzzler.*

Answer: With a $200 monthly payment, the remaining discretionary income is $400 — leaving us with a budget constraint of $0.2x_1 + x_2 = 400$. Solving the optimization problem

$$\max_{x_1,x_2} x_1^{0.5}x_2^{0.5} \text{ subject to } 400 = 0.2x_1 - x_2, \tag{7.19}$$

we get $x_1 = 1000$ and $x_2 = 200$. Thus, you would drive 1000 miles per month.

(b) *Suppose you are indifferent between the gas-guzzler and the fuel efficient car. How much must the monthly payment for the fuel efficient car be?*

Answer: The utility you receive with the gas guzzler is $1000^{0.5}200^{0.5} \approx 447.2$. To be indifferent between the gas guzzler and the fuel efficient car, we first calculate the bundle that you must be consuming if all your income after paying the higher monthly payment is spent; i.e. we solve

$$\min_{x_1,x_2} 0.1x_1 + x_2 \text{ subject to } x_1^{0.5}x_2^{0.5} = 447.2. \tag{7.20}$$

Solving the first two first order conditions, we get $x_2 = 0.1x_1$. Plugging this into the constraint and solving for x_1, we get $x_1 \approx 1414.21$, and plugging this back into $x_2 = 0.1x_1$ we get $x_2 \approx 141.42$. Thus, what we usually call bundle B is (1414.21,141.42). This bundle costs $0.1(1414.21) + 141.42 = 282.84$. Since we start with an income of $600, this implies that the monthly payment for the fuel efficient car is $600 – $282.84 = $317.16.

(c) *Now suppose that the government imposes a tax on gasoline that doubles the price per mile driven of each of the two cars. Calculate the optimal consumption bundle under each of the new budget constraints.*

Answer: These will be $(x_1,x_2) = (500,200)$ with the gas guzzler and $(x_1,x_2) = (707.11,141.42)$ with the fuel efficient car. (You can solve for this by setting up the optimization problem again. Alternatively, you can recognize that, with Cobb-Douglas tastes, consumption of each good is independent of the price of the other. Thus, consumption of x_2 remains as before since the price of x_2 has not changed — which means the remainder is spent on x_1. Thus, miles driven falls by half for each type of car.)

(d) *Do you now switch to the fuel efficient car?*

Answer: Given that we just calculated the optimal bundle for each car type, we can calculate the utility you will get under each budget. Plugging (500,200) in the utility function, we get $u = 316.23$ under the gas guzzling budget; and plugging in (707.11,141.42), we also get $u = 316.23$. Thus, you will still be indifferent between the two car types.

(e) *Consider the utility function you have worked with so far as a special case of the CES family $u(x_1,x_2) = (0.5x_1^{-\rho} + 0.5x_2^{-\rho})^{-1/\rho}$. Given what you concluded in A(d) of this question, how would your answer to B(d) change as ρ changes?*

Answer: We concluded in part A that we are more likely to switch to the fuel efficient car the more complementary “miles driven” (x_1) is to “other consumption” (x_2) — and more likely to switch to the gas guzzler the more substitutable these goods are. As it turns out, our calculations here have demonstrated that Cobb-Douglas tastes (with $\rho = 0$ and elasticity of substitution equal to 1) encompass exactly that amount of substitutability that will keep us indifferent between the gas guzzler and the fuel efficient car as the price of gasoline increases. For elasticities of substitution greater than 1 — i.e. for $-1 \leq \rho < 0$ — we would switch to the gas guzzler; for elasticities less than 1 — i.e. for $\rho > 0$, we would switch to the fuel efficient car.

Conclusion: Potentially Helpful Reminders

1. *Important Graphing Hint*: When graphing income and substitution effects, it is very helpful to draw the original indifference curve with lots of substitutability — i.e. with relatively little curvature — unless specifically told to do otherwise. If you do this, it becomes much harder to trick yourself into thinking that something which is logically impossible is actually happening in your graphs.

2. Keep in mind the following: Substitution effects always occur *along a single indifference curve* and income effects always involve *jumping from one indifference curve to another across two parallel budgets*.

3. Since concepts like homotheticity, quasilinearity, normal and inferior goods, and luxuries and necessities are definitions about how indifference curves within an indifference map relate to one another, they are relevant only for determining income effects. In fact, we can get both large and small substitution effects for any of these types of tastes and goods — with the size of the substitution effect depending on the curvature of the original indifference curve (which has no relation to whether goods are normal or inferior or homothetic, etc.).

4. In the text, we emphasize the more common of the two types of substitution effects that economists talk about — the effect that holds “real welfare” fixed and thus occurs along an indifference curve. This effect is also called the *Hicks substitution effect* and it differs from a second type of substitution effect (called the *Slutsky substitution effect*) that assumes a consumer is compensated enough to afford the original bundle (rather than to reach the original indifference curve). This second type of substitution effect is almost identical to the first, particularly for small changes in prices — and it appears in end-of-chapter exercises 7.6 and 7.11 for you to explore.

5. Often students confuse Giffen goods with a certain type of “prestige good” that people value more as it gets more expensive. That is definitely not what a Giffen good is — and you can do end-of-chapter exercise 7.9 to work through the difference between these two types of goods.

6. The math (in part B of *Microeconomics: An Intuitive Approach with Calculus*) follows straightforwardly from the graphical intuitions: Maximize utility subject to budget constraints to get what people do at bundles *A* and *C* (when income and substitution effects are combined) — but minimize the expenditure it takes to get to the original utility level at the new prices to find *B* (and thus the substitution effect).