CHAPTER

8

Wealth and Substitution Effects in Labor and Capital Markets

We introduced income and substitution effects in Chapter 7 and now extend those concepts to budgets that arise from endowments. The most important such budgets involve labor/leisure choices (by workers) and intertemporal consumption choices (by savers and borrowers). What we called "income effects" in Chapter 7 now become "wealth effects" — but the substitution effects remain exactly the same. In principle there is nothing new in this chapter — but it will become clearer why it will generally not work to simply try to memorize which way income (or wealth) effects and substitution effects point for different goods. A much better strategy is to understand the concepts and then be able to apply them to all possible circumstances you might encounter.

Chapter Highlights

The main points of the chapter are:

1. When you own your current consumption bundle, a **price change in either direction makes you better off** (assuming low transactions costs).

2. **The same is not true if** your budget arises from an endowment but **your optimal choice is not the endowment bundle**. If you are a net seller of a good, then an increase in the price of the good makes you better off. If you are a net buyer, an increase in the price might make you better off or worse off depending on the degree of substitutability between the goods.

3. The **substitution effect looks exactly the same way** regardless of whether the budget is exogenous or whether it arises from an endowment. The "wealth" effect, however, may point in a different direction.

4. Under reasonable assumptions about the underlying goods, wealth and substitution effects point in different directions when choices involve labor or

savings decisions. As a result **labor responses to wage changes and savings responses to interest rate changes are ambiguous**. This points to the importance of understanding the degree of substitutability between goods as one thinks about labor and savings responses to price changes.

5. The underlying mathematics developed in part B of the chapter is exactly the same for deriving substitution effects, with the only difference emerging as we derive the initial and final optimal bundles where we have to use the endowment-generated budget constraint rather than the previous exogenous budget constraint (in Chapter 7).

Using the LiveGraphs

For an overview of what is contained on the LiveGraphs site for each of the chapters (from Chapter 2 through 29) and how you might utilize this resource, see pages 2-3 of Chapter 1 of this *Study Guide*. To access the LiveGraphs for Chapter 8, click the *Chapter 8* tab on the left side of the LiveGraphs web site.

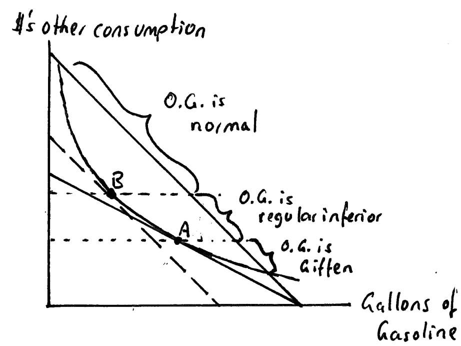
Since this chapter covers the same basic concepts as Chapter 7, we have not at this point developed further *Exploring Relationships* modules for this chapter. Be sure, though, to check out the ones in Chapter 7.

8A Solutions to Within-Chapter-Exercises for Part A

Exercise 8A.1 *Since George's situation is equivalent to a decrease in the price of other goods (with exogenous income), illustrate where on his final budget George would consume if other goods are normal, regular inferior and Giffen.*

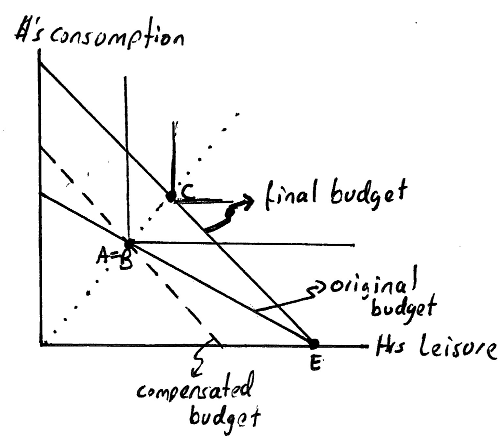
Answer: This is done in Graph 8.1 (on the next page). Other goods are Giffen if a decrease in their price leads to less consumption than originally at A. They are regular inferior if they are not Giffen but an increase in income causes a decrease in consumption. The increase in income is seen from the compensated to the final budget — so if consumption falls between A and B, other goods are regular inferior. Finally, other goods are normal if an increase in income (from the compensated budget to the final budget) results in an increase in consumption (from B).

Exercise 8A.2 *Illustrate substitution and wealth effects — i.e. the initial bundle, the bundle that incorporates a substitution effect from a wage increase, and the final bundle chosen under the wage increase — assuming that your tastes for consumption and leisure are properly modeled as perfect complements.*



Graph 8.1: Type of "other goods"

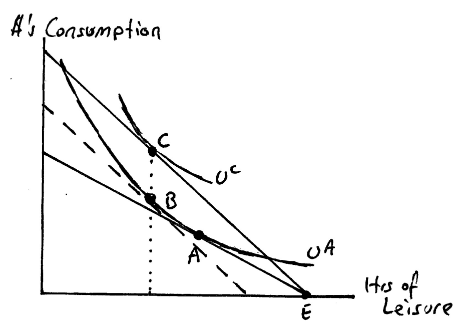
Answer: This is illustrated in Graph 8.2 where the substitution effect disappears ($A = B$) due to the perfect complementarity between consumption and leisure.



Graph 8.2: Wage Increase with Leisure and Consumption Perfect Complements

Exercise 8A.3 *Replicate the previous exercise under the assumption that your tastes are quasi-linear in leisure.*

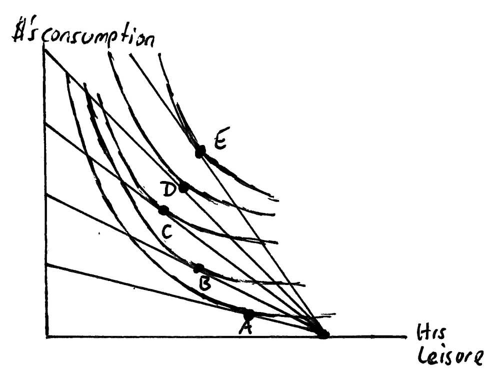
Answer: This is illustrated in Graph 8.3 (on the next page) where C lies above B because of the lack of an income effect due to the quasilinearity of tastes in leisure. Thus, with respect to leisure, there is no wealth effect.



Graph 8.3: Wage Increase when Leisure is Quasilinear

Exercise 8A.4 *Illustrate a set of indifference curves that gives rise to the kind of response to wage changes as described.*

Answer: This is illustrated in Graph 8.4. At the lowest wage in the graph, *A* is optimal — and entails relatively little labor (and a lot of leisure). As the wage increases, *B* becomes optimal — with less leisure and more labor. Similarly, labor supply increases as the wage increases further and *C* becomes optimal. But then, as wage increases again, *D* is optimal — and involves less labor and more leisure than *C*. This continues at *E* where labor supply again falls as wage increases.



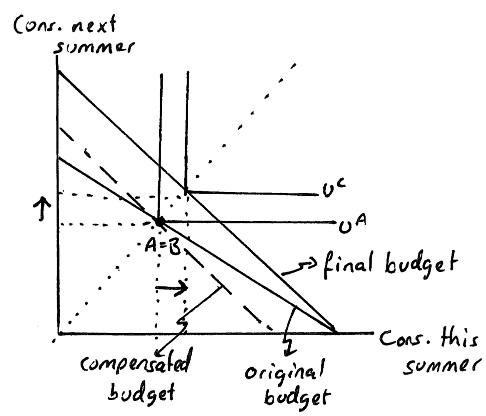
Graph 8.4: Tastes for which labor supply initially increases and then decreases as wage goes up

Exercise 8A.5 True or False*: For decreases in wage taxes, substitution effects put positive pressure on tax revenues while wealth effects typically put negative pressure on revenues.*

Answer: This is true. When wage taxes fall, this is (from the workers' perspective) equivalent to an increase in their (take-home) wage. An increase in the wage increases the opportunity cost of consuming leisure — which causes the substitution effect to point in the direction of less leisure, more labor. An increase in the number of hours worked would put upward pressure on tax revenues. The wealth effect of a wage increase, however, typically points in the opposite direction — at least so long as leisure is a normal good. Thus, the wealth effect of a tax decrease causes people to take more leisure and work less — which would put downward pressure on tax revenues from wage taxes.

Exercise 8A.6 *Illustrate that your savings will decline with an increase in the interest rate if consumption this summer and next summer are perfect complements.*

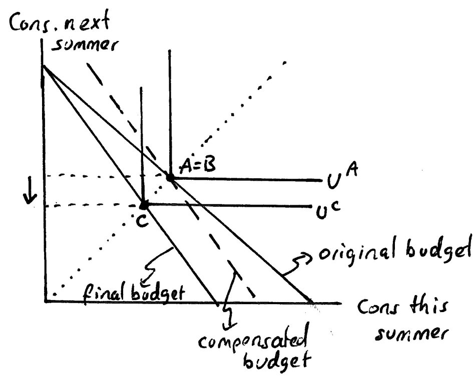
Answer: Graph 8.5 illustrates that consumption in both periods will increase as a result of the increase in the interest rate — with the entire change due to a wealth effect (given that the substitution effect disappears with perfect complements; i.e. given A=B.) Since consumption this summer increases, the amount you are putting into your savings account decreases.



Graph 8.5: Increasing Interest Rate and Savings

Exercise 8A.7 *Illustrate how consumption next summer changes with an increase in the interest rate if consumption this summer and next summer are perfect complements (and all your income occurs next summer).*

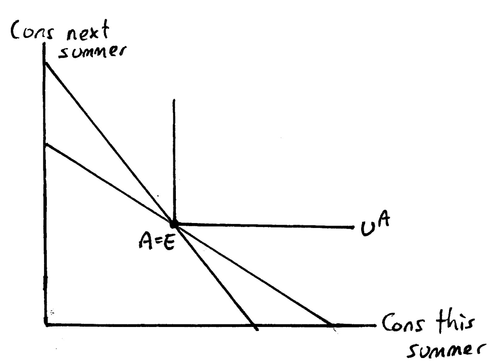
Answer: Graph 8.6 (next page) illustrates that consumption next summer will unambiguously decline because the only remaining effect is the wealth effect.



Graph 8.6: Increasing Interest Rate and Borrowing

Exercise 8A.8 *Demonstrate that the only way you will not violate Shakespeare's advice as the interest rate goes up is if consumption this summer and next are perfect complements.*

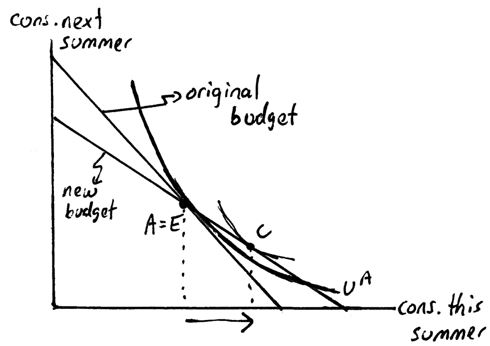
Answer: If there is any curvature in the indifference curve at *A* in Graph 8.7, then the new (steeper) budget line will cut the indifference curve from above and will thus make "better" bundles available (which will involve less consumption now; i.e. savings.) Thus, the only way we would not violate Shakespeare's advice to "neither borrow nor lend" is if there was a sufficiently large kink at *A* such that *A* once again is optimal on the new (steeper) budget. This is certainly the case when consumption now and consumption in the future are perfect complements. (It is technically also true for indifference curves with less extreme kinks which technically don't represent perfect complements.)



Graph 8.7: Sticking by Shakespeare's Advice

Exercise 8A.9 *Illustrate that (unless consumption this summer and consumption next summer are perfect complements) you will violate the first part of Shakespeare's advice — not to be a borrower — if the interest rate fell instead of rose.*

Answer: This is illustrated in Graph 8.8 where the initial budget constraint is the one with steeper slope (i.e. higher interest rate), with A=E optimal where u^A is tangent to the original budget. The new budget with shallower slope (i.e. lower interest rate) then cuts the original budget at E — which implies it cuts the original indifference curve u^A from below. That in turn opens a number of new bundles that lie above u^A — all of which lie to the right of A. (One example of a possible new optimum is C). Thus, you will consume more now — which means you will borrow.



Graph 8.8: Violating Shakespeare in the other direction

8B Solutions to Within-Chapter-Exercises for Part B

Exercise 8B.1 *With the numbers in the previous paragraph, George's income is \$2,000 per week. Verify that you would get the same optimal consumption bundle if you modeled this as a constrained optimization problem in which income was exogenously set at \$2,000 per week.*

Answer: In that case, the budget constraint would be $2x_1 + x_2 = 2000$. We would then solve the problem

$$\max_{x_1,x_2} \; x_1^{0.1}x_2^{0.9} \; \text{ subject to } \; 2x_1 + x_2 = 2000. \tag{8.1}$$

The Lagrange function for this problem is

$$\mathcal{L}(x_1, x_2, \lambda) = x_1^{0.1} x_2^{0.9} + \lambda(2000 - 2x_1 - x_2). \tag{8.2}$$

The first two first order conditions for this problem are then

$$\frac{\partial \mathcal{L}}{\partial x_1} = 0.1 x_1^{-0.9} x_2^{0.9} - 2\lambda = 0,$$
$$\frac{\partial \mathcal{L}}{\partial x_2} = 0.9 x_1^{0.1} x_2^{-0.1} - \lambda = 0. \tag{8.3}$$

Taking the λ terms to the right hand side and then dividing one equation by the other (and thus eliminating λ), we can solve for x_2 in terms of x_1 to get $x_2 = 18x_1$. Plugging this back into the budget constraint, we get $2x_1 + 18x_1 = 2000$ or $x_1 = 100$. Finally, plugging this back into $x_2 = 18x_1$ gives us $x_2 = 1800$.

Exercise 8B.2 *Verify that the above solutions are correct.*

Answer: The Lagrange function for this problem is

$$\mathcal{L}(x_1, x_2, \lambda) = 4x_1 + x_2 + \lambda(1348 - x_1^{0.1} x_2^{0.9}). \tag{8.4}$$

The first two first order conditions for this problem are then

$$\frac{\partial \mathcal{L}}{\partial x_1} = 4 - 0.1\lambda x_1^{-0.9} x_2^{0.9} = 0,$$
$$\frac{\partial \mathcal{L}}{\partial x_2} = 1 - 0.9\lambda x_1^{0.1} x_2^{-0.1} = 0. \tag{8.5}$$

Adding the negative terms to both sides and then dividing the two equations by one another, we can eliminate the λ terms and can solve for $x_2 = 36x_1$. Plugging this into the constraint, we get $x_1^{0.1}(36x_1)^{0.9} = 1348$ which solves to $x_1 = 53.58$. Plugging this back into $x_2 = 36x_1$, we also get $x_2 \approx 1929$.

Exercise 8B.3 *How much (negative) compensation was required to get George to be equally well off when the price of gasoline increased?*

Answer: The bundle $(53.58, 1929)$ costs $4(53.58) + 1929 \approx 2143$. At a price of \$4 per gallon, George's 1000 gallons per week are worth \$4,000 per week. Since he only needs \$2,143 to remain as happy as he was when the price per gallon was \$2, the necessary compensation is approximately $-\$1,857$.

Exercise 8B.4 *Solve the problem defined in equation (8.11).*

Answer: The Lagrange function is

$$\mathcal{L}(c, \ell, \lambda) = c + 25\ell + \lambda(1998 - c - 400 \ln \ell). \tag{8.6}$$

The first two first order conditions for this problem are then

$$\frac{\partial \mathscr{L}}{\partial c} = 1 - \lambda = 0,$$
$$\frac{\partial \mathscr{L}}{\partial \ell} = 25 - \frac{400\lambda}{\ell} = 0. \tag{8.7}$$

Solving these, we get $\ell = 16$. Plugging this back into the constraint $c + 400 \ln \ell = 1998$, we get $c \approx 889$.

Exercise 8B.5 *Suppose your tastes were more accurately modeled by the Cobb–Douglas utility function $u(c,\ell) = c^{0.5}\ell^{0.5}$. Determine wealth and substitution effects — and graph your answer.*

Answer: First, we can determine the bundles A and C — the initial and final bundles — by solving the usual maximization problem

$$\max_{c,\ell}\ c^{0.5}\ell^{0.5} \ \text{ subject to } \ c = w(60-\ell). \tag{8.8}$$

The Lagrange function for this problem is

$$\mathscr{L}(c,\ell,\lambda) = c^{0.5}\ell^{0.5} + \lambda(60w - w\ell - c). \tag{8.9}$$

The first two first order conditions for this problem are then

$$\frac{\partial \mathscr{L}}{\partial c} = 0.5c^{-0.5}\ell^{0.5} - \lambda = 0,$$
$$\frac{\partial \mathscr{L}}{\partial \ell} = 0.5c^{0.5}\ell^{-0.5} - \lambda w = 0. \tag{8.10}$$

Solving these to eliminate λ, we get $c = w\ell$. Substituting into the budget constraints, $c = w(60-\ell)$, we get $w\ell = w(60-\ell)$ which solves to $\ell = 30$. Finally, plugging this back into $c = w(60-\ell)$, we get $c = 30w$.

At the initial wage of $w = 20$, the optimal bundle A is therefore $(c,\ell) = (600,30)$. At the new wage $w = 25$, the optimal bundle C is $(c,\ell) = (750,30)$.

To decompose this into substitution and wealth effects, we need to determine bundle B — the bundle we would consume if we only faced a change in opportunity costs but a change in wealth that kept us on the same indifference curve. First, we need to calculate the utility at A — which is $u^A = u(600,30) = 600^{0.5}30^{0.5} \approx 134.164$. We then solve the problem

$$\min_{c,\ell}\ c + 25\ell \ \text{ subject to } \ c^{0.5}\ell^{0.5} = 134.164. \tag{8.11}$$

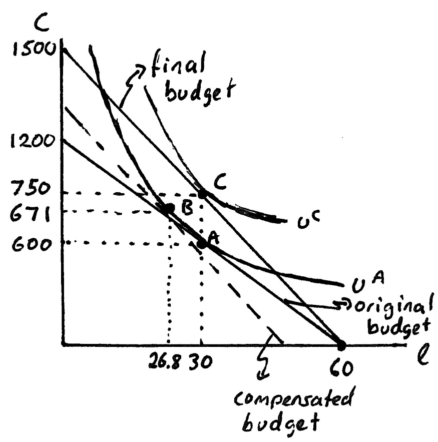
The Lagrange function for this problem is

$$\mathscr{L}(c,\ell,\lambda) = c + 25\ell + \lambda(134.164 - c^{0.5}\ell^{0.5}). \tag{8.12}$$

The first two first order conditions for this problem are then

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c} &= 1 - 0.5\lambda c^{-0.5}\ell^{0.5} = 0,\\ \frac{\partial \mathcal{L}}{\partial \ell} &= 25 - 0.5\lambda c^{0.5}\ell^{-0.5} = 0.\end{aligned} \tag{8.13}$$

Solving these to eliminate λ, we get $c = 25\ell$. Plugging this into the constraint, we get $(25\ell)^{0.5}\ell^{0.5} = 134.164$ which reduces to $\ell \approx 26.83$. Finally, substituting back into $c = 25\ell$, we can derive $c \approx 670.82$. Thus, bundle B is $(c,\ell) = (26.83, 670.82)$ — and the movement from A to B is the substitution effect while the movement from B to C is the wealth effect. This is depicted in Graph 8.9. In terms of leisure, the substitution and wealth effects are directly offsetting.



Graph 8.9: Wage Increase with Cobb-Douglas Tastes

Exercise 8B.6 *What is the equation for the Laffer Curve in Graph 8.10?*

Answer: This is given by the tax rate t times the wage income which is the wage w times the amount of labor provided; i.e.

$$t\left[25\left(60 - \frac{400}{25(1-t)}\right)\right] = t\left(1500 - \frac{400}{(1-t)}\right) = 1500t - \frac{400t}{(1-t)} \tag{8.14}$$

Exercise 8B.7 *Solve for the peak of the Laffer Curve (using the equation you derived in the previous exercise) and verify that it occurs at a tax rate of approximately 48.4%.*

Answer: The peak occurs where the derivative of the function is equal to zero; i.e. where

$$1500 - \frac{400}{(1-t)} - \frac{400t}{(1-t)^2} = 0. \tag{8.15}$$

Dividing though by 100 and multiplying by $(1-t)^2$, this turns into

$$15(1-t)^2 - 4(1-t) - 4t = 0. \tag{8.16}$$

Then, multiplying out the terms and combining like terms, we get

$$15t^2 - 30t + 11 = 0. \tag{8.17}$$

The quadratic formula tells us that any equation of the form $ax^2+bx+c=0$ has two solutions given by

$$x = \frac{-b+(b^2-4ac)^{0.5}}{2a} \text{ and } x = \frac{-b-(b^2-4ac)^{0.5}}{2a}. \tag{8.18}$$

In our equation, $x = t$, $a = 15$, $b = -30$ and $c = 11$. The two solutions given by the quadratic formula are then

$$\begin{aligned} t &= \frac{30-(30^2-4(15)(11))^{0.5}}{2(15)} \approx 0.4836 \text{ and} \\ t &= \frac{30+(30^2-4(15)(11))^{0.5}}{2(15)} \approx 1.517. \end{aligned} \tag{8.19}$$

The latter occurs outside the economically relevant range of possible tax rates (which can range from 0 to 1) — which leaves us with the first solution that verifies what is graphed in the text.

Exercise 8B.8 *Verify that this is indeed the solution to the problem defined in (8.15).*

Answer: The Lagrange function for this problem is

$$\mathscr{L}(c_1, c_2, \lambda) = c_1^{0.5} c_2^{0.5} + \lambda(10000(1+r) - (1+r)c_1 - c_2). \tag{8.20}$$

The first two first order conditions for this problem are then

$$\begin{aligned} \frac{\partial \mathscr{L}}{\partial c_1} &= 0.5c_1^{-0.5}c_2^{0.5} - \lambda(1+r) = 0, \\ \frac{\partial \mathscr{L}}{\partial c_2} &= 0.5c_1^{0.5}c_2^{-0.5} - \lambda = 0. \end{aligned} \tag{8.21}$$

These solve to $c_2 = (1+r)c_1$. Plugging this into the budget constraint, we get $10000(1+r) = (1+r)c_1 + (1+r)c_1$ which we can solve to get $c_1 = 5000$. Plugging this back into $c_2 = (1+r)c_1$, we can also solve for $c_2 = 5000(1+r)$.

Exercise 8B.9 *Verify that this is indeed the solution to the problem defined in (8.17).*

Answer: The Lagrange function for this problem is

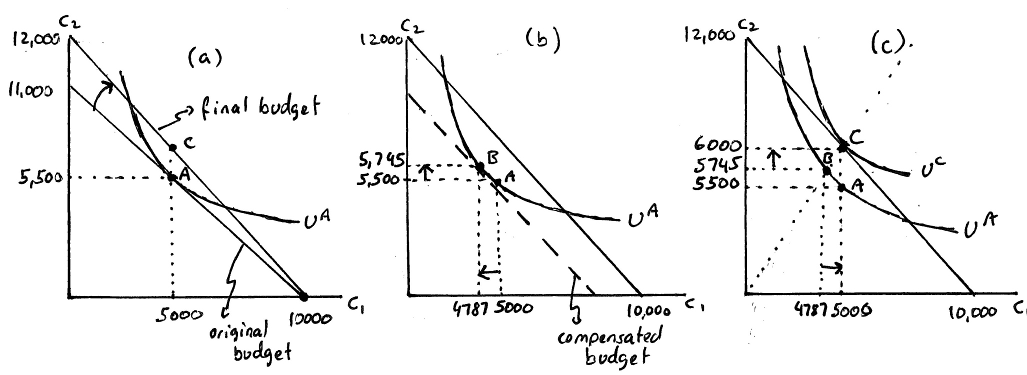
$$\mathscr{L}(c_1, c_2, \lambda) = (1+r)c_1 + c_2 + \lambda\left(5244 - c_1^{0.5} c_2^{0.5}\right). \tag{8.22}$$

The first two first order conditions for this problem are then

$$\begin{aligned} \frac{\partial \mathscr{L}}{\partial c_1} &= (1+r) - 0.5\lambda c_1^{-0.5} c_2^{0.5} = 0, \\ \frac{\partial \mathscr{L}}{\partial c_2} &= 1 - 0.5\lambda c_1^{0.5} c_2^{-0.5} = 0. \end{aligned} \tag{8.23}$$

Solving these to eliminate λ, we again get $c_2 = (1+r)c_1$. Plugging this into the constraint, we get $c_1^{0.5}[(1+r)c_1]^{0.5} = 5244$ which solves to $c_1 = 5244/(1+r)^{0.5}$. Plugging this back into $c_2 = (1+r)c_1$, we also get $c_2 = 5244(1+r)^{0.5}$. When $r = 0.2$, this implies that $c_1 = 5244/(1+0.2)^{0.5} \approx 4{,}787.1$ and $c_2 = 5244(1+0.2)^{0.5} \approx 5{,}744.5$. (The answers are off slightly from what is derived in the text because we used the rounded value 5244 for utility.)

Exercise 8B.10 *Using a set of graphs similar to those depicted in Graph 8.5, label the bundles that we have just calculated.*

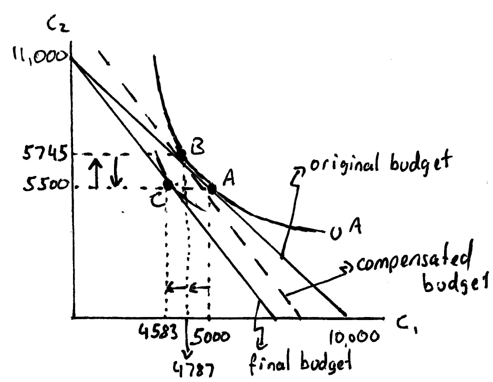


Graph 8.10: Interest Rate Increase and Saving with Cobb-Douglas Tastes

Answer: This is done in panels (a) through (c) of Graph 8.10. Panel (a) indicates the original optimal bundle A=(5000,5500) and the change in the intertemporal budget constraint with an increase in the interest rate. The final optimum C=(5000,6000) is also anticipated in panel (a). Panel (b) focuses on the substitution effect by illustrating the compensated budget tangent to the original indifference curve at B=(4787,5745), with the arrows indicating the substitution effect relative to consumption in each period. (The effect, as always, moves us away from consumption where it has become more expensive and toward where it has become cheaper.) Finally, panel (c) illustrates (again with arrows) the wealth effect that moves us from the original indifference curve u^A to the final indifference curve u^C. Since consumption in both periods is a normal good, the wealth effect is positive for consumption in both periods — and exactly offsets the substitution effect with respect to current consumption.

Exercise 8B.11 *Illustrate what we have just calculated in a graph.*

Answer: This is done in Graph 8.11 where the wealth and substitution effects are offsetting on the vertical axis but point in the same direction on the horizontal.



Graph 8.11: Interest Rate Increase and Borrowing with Cobb-Douglas Tastes

Exercise 8B.12 *We calculated above that consumption next summer is unchanged as the interest rate rises when tastes can be represented by the Cobb-Douglas utility function we used. This is because this function assumes an elasticity of substitution of 1. How would this result change if the elasticity of substitution is larger or smaller than 1?*

Answer: If the elasticity of substitution is greater than 1, then the substitution effect increases in magnitude and will dominate — implying that consumption next summer increases as the interest rate rises. If, on the other hand, the elasticity of substitution is less than 1, then the substitution effect decreases in magnitude and will be dominated by the wealth effect — implying that consumption next summer decreases as the interest rate rises.

Exercise 8B.13 *Verify that (8.22) and (8.33) are correct.*

Answer: The maximization problem is

$$\max_{c_1,c_2} \; c_1^{0.5}c_2^{0.5} \; \text{ subject to } \; (1+r)c_1+c_2=5000(1+r)+5500. \tag{8.24}$$

This results in the Lagrange function

$$\mathcal{L}(c_1,c_2,\lambda)=c_1^{0.5}c_2^{0.5}+\lambda(5000(1+r)+5500-(1+r)c_1-c_2). \tag{8.25}$$

The first two first order conditions for this problem are then

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial c_1}&=0.5c_1^{-0.5}c_2^{0.5}-\lambda(1+r)=0,\\ \frac{\partial\mathcal{L}}{\partial c_2}&=0.5c_1^{0.5}c_2^{-0.5}-\lambda=0.\end{aligned} \tag{8.26}$$

As before, this solves to $c_2=(1+r)c_1$. Substituting this into the constraint, we get $5000(1+r)+5500=(1+r)c_1+(1+r)c_1$ which solves to $c_1=2500+(2750/(1+r)$ and, substituting this back into $c_2=(1+r)c_1$, $c_2=2500(1+r)+2750$. At the interest rate $r=0.1$, we therefore start at $c_1=2500+(2750/1.1)=5000$ and $c_2=1.1(2500)+2750=5500$. When the interest rate rises to 0.2, $c_2=2500+(2750/1.2)=4791.67$ and $c_2=1.2(2500)+2750=5750$. We therefore move from A=(5000,5500) to $C=(4792,5750)$.

We find point B by first determining the utility at bundle A — i.e. we calculate that $u^A=5000^{0.5}5500^{0.5}\approx 5244$. We then solve the problem

$$\min_{x_1,x_2} \; 1.2c_1+c_2 \; \text{ subject to } \; c_1^{0.5}c_2^{0.5}=5244. \tag{8.27}$$

After writing down the Lagrange function, we can solve the first two first order conditions as before — giving us $c_2=1.2c_1$. Plugging this back into the constraint $c_1^{0.5}c_2^{0.5}=5244$, we get $c_1^{0.5}(1.2c_1)^{0.5}=5244$ which solves to $c_1\approx 4781.1$. Plugging this back into $c_2=1.2c_1$, we also get $c_2\approx 5744.5$. (The numbers taken to two decimals are slightly different than those in the text because we rounded when we set utility equal to 5244.)

End of Chapter Exercises

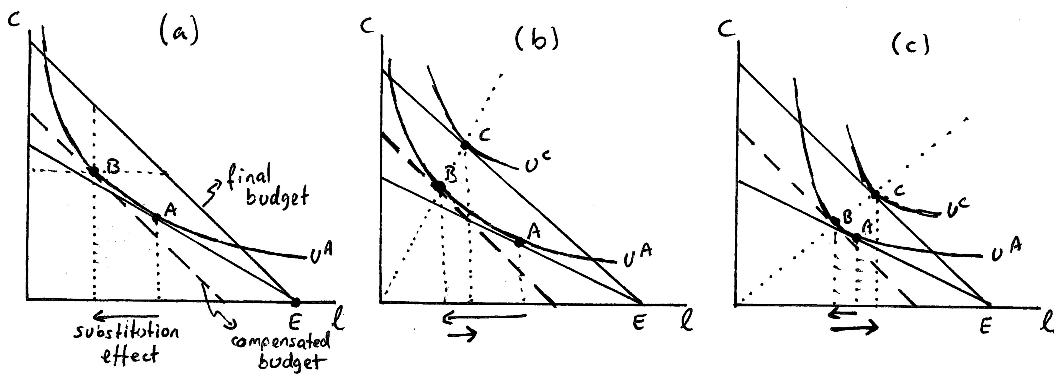
Exercise 8.1

As we have suggested in the chapter, it is often important to know whether workers will work more or less as their wage increases.

A: *In each of the following cases, can you tell whether a worker will work more or less as his wage increases?*

(a) *The worker's tastes over consumption and leisure are quasilinear in leisure.*

Answer: Panel (a) of Graph 8.12 (on the next page) illustrates the substitution effect for a wage increase. This effect depends only on the shape of the indifference curve that goes through the original bundle A — the more substitutable consumption and leisure are, the greater the substitution from leisure (and thus toward more labor) to consumption. If tastes are then quasilinear in leisure, we know that, as we move from the compensated to the final



Graph 8.12: Wage Increases with Different Tastes

budget, there is no wealth effect on leisure and thus no further change in leisure (beyond the substitution effect). Thus, the worker will unambiguously work more.

(b) *The worker's tastes over consumption and leisure are homothetic.*

Answer: Panels (b) and (c) of Graph 8.12 illustrate that it is ambiguous in this case whether the worker will work more or less with an increase in the wage — it depends on the size of the substitution effect. In panel (b), the indifference curve u^A is relatively flat around A — indicating a great deal of willingness on the part of the worker to substitute leisure and consumption. This gives rise to a large substitution effect. B is tangent to the (dashed) compensated budget — which is parallel to the final budget. Homotheticity then implies that, if B is optimal on the compensated budget, the optimal final bundle C lies on a ray from the origin through B. Because of the willingness to substitute between consumption and leisure, the resulting wealth effect only outweighs part of the substitution effect — leaving us with less leisure (and more labor) at the higher wage than at the original lower wage (at A). In panel (c), on the other hand, consumption and leisure are not as substitutable around A — leading to a relatively small substitution effect that is more than outweighed by a wealth effect in the opposite direction. Thus, when consumption and leisure are relatively complementary, an increase in the wage causes an increase in leisure and thus a decrease in work hours.

(c) *Leisure is a luxury good.*

Answer: We can use the same graphs as in panels (b) and (c) to again show that the answer is ambiguous. If leisure is a luxury good, then as the budget shifts out parallel, the new optimal bundle will lie to the right of the ray from the origin through the original optimum (because consumption of leisure increases faster than under homotheticity). In panel (b), that would mean C lies to the right of where it is indicated in the graph — but that still makes it plausible that the wealth effect is smaller than the substitution effect leaving us with less leisure than at A (and thus more work). In panel (c), C will again lie to the right of where it is indicated in the graph — but that implies that the wealth effect is even larger and will still outweigh the substitution effect. This will again leave us with more leisure and thus less work.

(d) *Leisure is a necessity.*

Answer: For reasons analogous to those just cited for luxury goods, the answer is still ambiguous and depends on the size of the substitution effect. This time, C will lie to the left of where it is marked in panels (b) and (c) of the graph — but that still leaves room for the ambiguity.

(e) *The worker's tastes over consumption and leisure are quasilinear in consumption.*

Answer: Going back to panel (a) of the graph, if consumption is the quasilinear good, then it will remain unchanged from the optimal bundle B on the compensated budget to the final budget. This creates a wealth effect on leisure that is opposite to the substitution effect. As drawn in panel (a), it looks like that returns us to a bundle C that will lie right above A — thus returning us to the same leisure consumption (and thus the same amount of work) as before the wage increase. But had we drawn a smaller substitution effect, the horizontal line through B would take us to the right of A on the final budget — thus causing an increase in leisure (and a decrease in work). If, on the other hand, we had made the indifference curve u^A flatter and thus had produced a larger substitution effect, the horizontal line through B would take us to the left of A on the final budget — thus causing a decrease in leisure (and thus an increase in work) from the original optimum A. As is usually the case when we have competing substitution and wealth effects, the answer is therefore again ambiguous.

B: *Suppose that tastes take the form* $u(c,\ell) = (0.5c^{-\rho} + 0.5\ell^{-\rho})^{-1/\rho}$.

(a) *Set up the worker's optimization problem assuming his leisure endowment is L and his wage is w.*

Answer: The problem is

$$\max_{c,\ell} \left(0.5c^{-\rho} + 0.5\ell^{-\rho}\right)^{-1/\rho} \text{ subject to } w(L-\ell) = c. \tag{8.28}$$

(b) *Set up the Lagrange function corresponding to your maximization problem.*

Answer: The Lagrange function is

$$\mathscr{L}(c,\ell,\lambda) = \left(0.5c^{-\rho} + 0.5\ell^{-\rho}\right)^{-1/\rho} + \lambda(wL - w\ell - c). \tag{8.29}$$

(c) *Solve for the optimal amount of leisure.*

Answer: The first two first order conditions are

$$\begin{aligned}
\frac{\partial \mathscr{L}}{\partial c} &= 0.5c^{-(\rho+1)}\left(0.5c^{-\rho} + 0.5\ell^{-\rho}\right)^{-(\rho+1)/\rho} - \lambda = 0,\\
\frac{\partial \mathscr{L}}{\partial \ell} &= 0.5\ell^{-(\rho+1)}\left(0.5c^{-\rho} + 0.5\ell^{-\rho}\right)^{-(\rho+1)/\rho} - \lambda w = 0.
\end{aligned} \tag{8.30}$$

The problem simplifies quite a bit if we simply take the λ terms to the other side of each equation and then divide the second equation by the first — which gives

$$\left(\frac{c}{\ell}\right)^{(\rho+1)} = w. \tag{8.31}$$

If you remember the expression of the MRS for a CES utility function from Chapter 5, you could have just skipped to this equation — which simply says the MRS is equal to the slope of the budget. The equation can then be written in terms of just $c = \ell w^{1/(\rho+1)}$. When plugged into the budget constraint $w(L-\ell) = c$, we can solve for

$$\ell = \frac{L}{1 + w^{-\rho/(\rho+1)}}. \tag{8.32}$$

(d) *Does leisure consumption increase or decrease as w increases? What does your answer depend on?*

Answer: We can see whether leisure increases or decreases with the wage rate by checking whether the first derivative of the equation for optimal leisure consumption from above is positive or negative. This derivative is (after a little algebra)

$$\frac{\partial \ell}{\partial w} = \rho\left[\frac{L\left(1 + w^{-\rho/(\rho+1)}\right)^{-2}}{(\rho+1)w^{(2\rho+1)/(\rho+1)}}\right] \tag{8.33}$$

Note L and w are positive and, since ρ lies between -1 and ∞, $(\rho+1)$ is also positive. This implies that the entire term in the square brackets must be positive regardless of what value ρ takes. (The negatives in the exponents of course only affect whether the term appears in the numerator or denominator — not whether it is positive or not.) Since the bracketed term is positive, the sign of the derivative depends entirely on whether ρ is positive or negative.

If $\rho = 0$, the tastes are Cobb-Douglas with elasticity of substitution $1/(1-\rho) = 1$. In that case, $\partial \ell / \partial w = 0$ and the wage therefore does not affect leisure consumption (or labor supply). For $\rho < 0$ the elasticity of substitution is greater than 1 — and $\partial \ell / \partial w < 0$. Thus, as the elasticity of substitution rises above 1, leisure consumption declines with an increase in the wage — and work hours increase. For $\rho > 0$, on the other hand, the elasticity of substitution is less than 1 — and $\partial \ell / \partial w > 0$. Thus, as the elasticity of substitution falls below 1, leisure consumption increases with the wage — and work hours fall. (Note that the elasticity of substitution is $\sigma = 1/(1+\rho)$.

(e) *Relate this to what you know about substitution and wealth effects in this type of problem.*

Answer: We have seen in part A of the question that the substitution effect points to less leisure (and more work) as wage increases — and, so long as leisure is a normal good, the wealth effect points in the opposite direction. For homothetic tastes (which CES tastes are), we showed that the overall effect of a wage increase on leisure consumption then depends on the substitutability of consumption and leisure. The greater the substitutability, the larger is the substitution effect — and the larger the substitution effect, the less likely it is that the wealth effect can fully offset it. We now see that for CES utility functions, the direction of the effect of a wage increase on leisure consumption depends entirely on ρ which determines the elasticity of substitution or the degree of substitutability between consumption and leisure. Elasticities below 1 make indifference curves look more like those in panel (c) of Graph 8.12 — with the wealth effect outweighing the substitution effect. Elasticities above 1, on the other hand, make the indifference curve look more like those in panel (b) where the substitution effect outweighs the wealth effect.

Exercise 8.5: *Savings Behavior and Tax Policy*

Policy Application: *Savings Behavior and Tax Policy: Suppose you consider the savings decisions of three households - households 1, 2 and 3. Each household plans for this year's consumption and next year's consumption, and each household anticipates earning \$100,000 this year and nothing next year. The real interest rate is 10%. Assume throughout that consumption is always a normal good.*

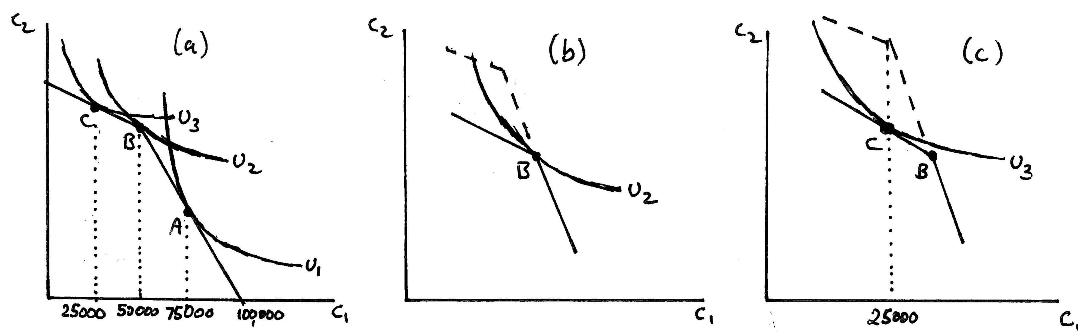
A: *Suppose the government does not impose any tax on interest income below \$5,000 but taxes any interest income above \$5,000 at 50%.*

(a) *On a graph with "Consumption this period" (c_1) on the horizontal axis and "Consumption next period" (c_2) on the vertical, illustrate the choice set faced by each of the three households.*

Answer: Panel (a) of Graph 8.13 (on the next page) illustrates the shape of the budget constraint which has a kink at \$50,000 of consumption now (c_1) because, when consumption now is \$50,000, then savings is also \$50,000 — which, at a 10% interest rate, results in \$5,000 of interest income. This first \$5,000 of interest income is exempt — which means the slope of the lower part of the budget constraint is simply $-(1+r) = -(1+0.1) = -1.1$. At current consumption below \$50,000, however, savings are above \$50,000 — which means interest income is above \$5,000. Thus, as interest income goes above \$5,000 at \$50,000 of savings, the slope of the budget constraint becomes shallower because the government now taxes the additional interest income at 50%. To be specific, the slope goes to $-(1+0.5r) = -1.05$.

(b) *Suppose you observe that household 1 saves \$25,000, household 2 saves \$50,000 and household 3 saves \$75,000. Illustrate indifference curves for each household that would make these rational choices.*

Answer: Panel (a) of the graph also indicates three indifference curves that make the choices of the 3 households optimal ones — with each indifference curve labeled by the relevant household. (For instance, u_1 refers to the optimal indifference curve from household 1's indifference map, where household 1 is the household that saves \$25,000 and thus consumes \$75,000 now.)



Graph 8.13: Savings of 3 Households

(c) *Now suppose the government changes the tax system by exempting the first \$7,500 rather than the first \$5,000 from taxation. Thus, under the new tax, the first \$7,500 in interest income is not taxed, but any interest income above \$7,500 is taxed at 50%. Given what you know about each household's savings decisions before the tax change, can you tell whether each of these households will now save more? (*Note*: It is extremely difficult to draw the scenarios in this question to scale — and when not drawn to scale, the graphs can become confusing. It is easiest to simply worry about the general shapes of the budget constraints around the relevant decision points of the households that are described.)*

Answer: This policy change would extend the steep portion of the budget from \$50,000 in current consumption to \$25,000 in current consumption (where savings hits \$75,000 and thus interest income hits \$7,500). Household 1 would be unaffected by this change since the indifference curve u_1 that is tangent at A lies above any new bundle that becomes available as a result of the policy change. Thus, household 1's savings would not change.

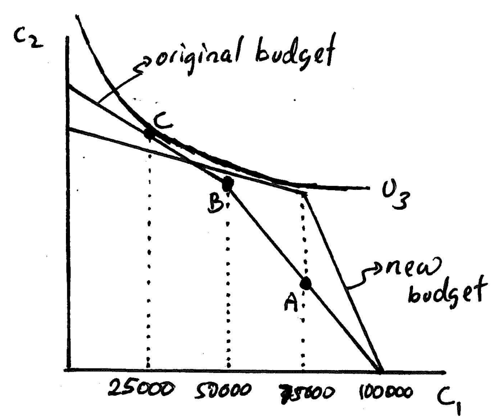
Household 2's savings, on the other hand, would almost certainly increase. In order for B to be optimal before the policy change, this household has an indifference curve that "hangs" on the kink of the original budget constraint. That means the MRS could lie between -1.1 (which is the slope of the steep portion of the budget) and -1.05 (which is the slope of the shallower portion). If the $MRS = -1.1$ at B, then the indifference curve u_2 is tangent to the extended steep budget that runs through B after the policy change — and thus B would continue to be optimal. However, if the MRS falls anywhere from -1.05 to -1.1 at B, then the new (dashed) budget constraint will cut the indifference curve u_2 as illustrated in panel (b) of Graph 8.13 — thus enabling the household to choose from a set of new bundles that lie above the original indifference curve. All of these bundles are such that consumption now (c_1) falls — i.e. savings increases.

Household 3, however, will definitely not save more. Panel (c) of the graph illustrates the change for this household. The new kink point now happens right above C. If the household were to choose a bundle on the flat portion of the new (dashed) budget line, then c_1 would be an inferior good and we have assumed that consumption is always normal. (It would be inferior because, when faced with a parallel outward shift in the budget, the household would be choosing to consume less.) Thus we know that the household will choose either the kink point (and keep savings the same) or a point on the steeper portion of the new (dashed) budget — with more c_1 and thus less savings.

(d) *Instead of the tax change in part (c), suppose the government had proposed to subsidize interest income at 100% for the first \$2,500 in interest income while raising the tax on any interest income above \$2,500 to 80%. (Thus, if someone earns \$2,500 in interest, she would receive an additional \$2,500 in cash from the government. If someone earns \$3,500, on the other hand,* *she would receive the same \$2,500 cash subsidy but would also have to pay \$800 in a tax.) One of the three households is overheard saying:"I actually don't care whether the old policy (i.e. the policy described in part A) or this new policy goes into effect." Which of the three households could have said this, and will that household save more or less (than under the old policy) if this new policy goes into effect?*

Answer: By subsidizing savings initially, the government in effect raises the interest rate from 10% to 20% for the first \$25,000 in savings. Thus, beginning at the \$100,000 intercept on the c_1 axis, the budget constraint is twice as steep. From that point on, however, the government is in effect reducing the interest rate from 10% to 2% because of the 80% tax on interest income. Thus, beginning at \$75,000 of current (c_1) consumption and moving leftward, the budget constraint becomes shallower than it was before. It seems clear that the two budget constraints will cross at some point — the question is where. We can check, for instance, which budget gives higher consumption next period (c_2) at \$50,000 of savings where the original kink occurred. Under the original policy, you make \$5,000 in interest when you save \$50,000 — giving you $c_2 = \$55,000$. Under the new policy, you get \$5,000 of interest (including the subsidy) for the first \$25,000 you save, you earn another \$2,500 of interest for the next \$25,000 in savings — but that is taxed at 80% to leave you with only \$500 of after-tax interest income. Thus, your total interest income (including the subsidy and subtracting out the tax) is \$5,500 — leaving you with \$55,500 in c_2. This is \$500 more than under the original policy. If you save an additional \$25,000 (for a total of \$75,000), you would earn an additional \$2,500 in interest. Under the original policy, half of that would be taxed away, leaving you with \$1,250. Under the new policy, 80% is taxed away — leaving you with only \$500 more. Thus, at \$75,000 of savings, the old policy results in greater c_2 than the new policy — \$250 more, to be exact. The old and the new budgets therefore intersect between \$75,000 and \$50,000 in savings.

The general relationship between the original and the new budget constraints is graphed in Graph 8.14. Households 1 and 2 must prefer the new policy since it opens up new bundles to the northeast of their original optimal bundles. Household 3, however, might be indifferent — as illustrated with the indifference curve u_3. Under the new policy, household 3 would then consume more now — and save less — if indeed it is indifferent between the policies.



Graph 8.14: Savings of 3 Households: Part II

B: *Now suppose that our 3 households had tastes that can be represented by the utility function $u(c_1,c_2) = c_1^{\alpha} c_2^{(1-\alpha)}$, where c_1 is consumption now and c_2 is consumption a year from now.*

(a) *Suppose there were no tax on savings income. Write down the intertemporal budget constraint with the real interest rate denoted r and current income denoted I (and assume that consumer anticipate no income next period).*

Answer: The intertemporal budget constraint is

$$(1+r)c_1 + c_2 = (1+r)I. \tag{8.34}$$

(b) *Write down the constrained optimization problem and the accompanying Lagrange function. Then solve for c_1, current consumption, as a function of α, and solve for the implied level of savings as a function of α, I and r. Does savings depend on the interest rate?*

Answer: The maximization problem is

$$\max_{c_1,c_2} c_1^{\alpha} c_2^{(1-\alpha)} \text{ subject to } (1+r)c_1 + c_2 = (1+r)I. \tag{8.35}$$

The Lagrange function for this problem is

$$\mathcal{L}(c_1, c_2, \lambda) = c_1^{\alpha} c_2^{(1-\alpha)} + \lambda((1+r)I - (1+r)c_1 - c_2). \tag{8.36}$$

The first two first order conditions can be solved to yield

$$c_2 = \frac{(1-\alpha)(1+r)}{\alpha} c_1. \tag{8.37}$$

Plugging this into the constraint $(1+r)c_1 + c_2 = (1+r)I$, we can solve for $c_1 = \alpha I$. Savings s is then simply c_1 subtracted from current income; i.e.

$$s = I - \alpha I = (1-\alpha)I. \tag{8.38}$$

Savings therefore does not depend on the interest rate.

(c) *Determine the α value for consumer 1 as described in part A.*

Answer: Consumer 1 saves 25% of her income on the portion of the budget where there is no tax — thus, it must be that $(1-\alpha) = 0.25$ or $\alpha = 0.75$.

(d) *Now suppose the initial 50% tax described in part A is introduced. Write down the budget constraint (assuming current income I and before-tax interest rate r) that is now relevant for consumers who end up saving more than \$50,000. (*Note*: Don't write down the equation for the kinked budget — write down the equation for the linear budget on which such a consumer would optimize.)*

Answer: To write down this budget, we need to know an intercept and a slope. The slope is simply $-(1+0.5r)$ since the government is taxing interest income at 50%. We can determine the c_2 intercept by calculating the total interest a consumer would earn if she saved all her income I assuming $I > 50,000$. For the first \$50,000, she would save at the untaxed interest rate of r — thus accumulating $(1+r)50000$ for next period. She would then have $(I - 50000)$ left to save — on which she would earn $0.5r$ interest. In addition to accumulating $(1+r)50000$ for the first \$50,000 in savings, she would therefore accumulate $(1+0.5r)(I-50000)$ if she saved all her income. Her total possible c_2 consumption is therefore

$$(1+r)50000 + (1+0.5r)(I-50000) = (1+0.5r)I + 25000r. \tag{8.39}$$

This, then, is the c_2 intercept. Given we already determined the slope to be $-(1+0.5r)$, the budget constraint is $c_2 = (1+0.5r)I + 25000r - (1+0.5)c_1$ or

$$(1+0.5r)c_1 + c_2 = (1+0.5r)I + 25000r. \tag{8.40}$$

(e) *Use this budget constraint to write down the constrained optimization problem that can be solved for the optimal choice given that households save more than \$50,000. Solve for c_1 and for the implied level of savings as a function of α, I and r.*

Answer: The maximization problem is

$$\max_{c_1,c_2} c_1^{\alpha} c_2^{(1-\alpha)} \text{ subject to } (1+0.5r)c_1 + c_2 = (1+0.5r)I + 25000r. \tag{8.41}$$

The Lagrange function for this problem is

$$\mathscr{L}(c_1,c_2,\lambda) = c_1^{\alpha} c_2^{(1-\alpha)} + \lambda((1+0.5r)I + 25000r - (1+0.5r)c_1 - c_2). \tag{8.42}$$

Solving this in the same way as before, we then get

$$c_1 = \alpha I + \frac{25000\alpha r}{(1+0.5r)} \tag{8.43}$$

and an implied savings s of

$$s = (1-\alpha)I - \frac{25000\alpha r}{(1+0.5r)}. \tag{8.44}$$

(f) *What value must α take for household 3 as described in part A?*

Answer: Household 3 saves \$75,000 with income of \$100,000 and before-tax interest rate $r = 0.1$. Thus

$$75000 = (1-\alpha)100000 - \frac{25000\alpha(0.1)}{(1+0.5(0.1))} \tag{8.45}$$

which solves to $\alpha \approx 0.244$.

(g) *With the values of α that you have determined for households 1 and 3, determine the impact that the tax reform described in (c) of part A would have?*

Answer: Using panels (b) and (c) of Graph 8.13, we concluded in part A that both households will choose to locate on the steeper portion of the budget under the new policy — i.e. on the portion defined by the constraint $(1+r)c_1 + c_2 = (1+r)I$ where $I = 100{,}000$ and $r = 0.1$. In B(b), we determined that savings in this case is given by $s = (1-\alpha)I$. Thus, household 1 for whom $\alpha = 0.75$ would save $(1-0.25)100{,}000 = 25{,}000$ as before. Household 3, for whom $\alpha \approx 0.244$, will save approximately $(1-0.244)100{,}000 = 75{,}600$ — but that level of savings lies to the left of the kink point of the dashed budget in panel (c). Thus, household 3 optimizes at the kink point, implying unchanged savings at \$75,000.

(h) *What range of values can α take for household 2 as described in part A?*

Answer: There are several ways you could use to figure this out. One way is to note that

$$MRS = -\frac{\alpha c_2}{(1-\alpha)c_1} \tag{8.46}$$

and that this must, for household 2, lie between -1.05 and -1.10 at $(c_1,c_2) = (50000, 55000)$ in order for that kink point in the budget to be optimal. Substituting these values for c_1 and c_2 into the expression for MRS and setting it equal to these two endpoint values, we can solve

$$-\frac{55000\alpha}{50000(1-\alpha)} = -1.05 \text{ and } -\frac{55000\alpha}{50000(1-\alpha)} = -1.10 \tag{8.47}$$

to conclude that $0.488 \le \alpha \le 0.5$.

Another way to solve for this is to use our results from the previous parts. Household 2 might have a tangency with the steep portion of the budget at $(c_1,c_2) = (50000, 55000)$. We concluded in B(b) (equation (8.38)) that in this case, savings s is $s = (1-\alpha)I$. Thus, for household 2 to choose \$50,000 in savings under the steeper portion of the budget, $50000 = (1-\alpha)100000$ which implies $\alpha = 0.5$.

Alternatively, household 2 could have a tangency with the shallow portion of the budget at $(c_1,c_2) = (50000, 55000)$. We concluded in B(3) that savings then satisfies equation (8.44). Substituting $s = 50{,}000$, $I = 100{,}000$ and $r = 0.1$ into that equation, we can solve for $\alpha \approx 0.488$. Thus, again we get that $0.488 \le \alpha \le 0.5$.

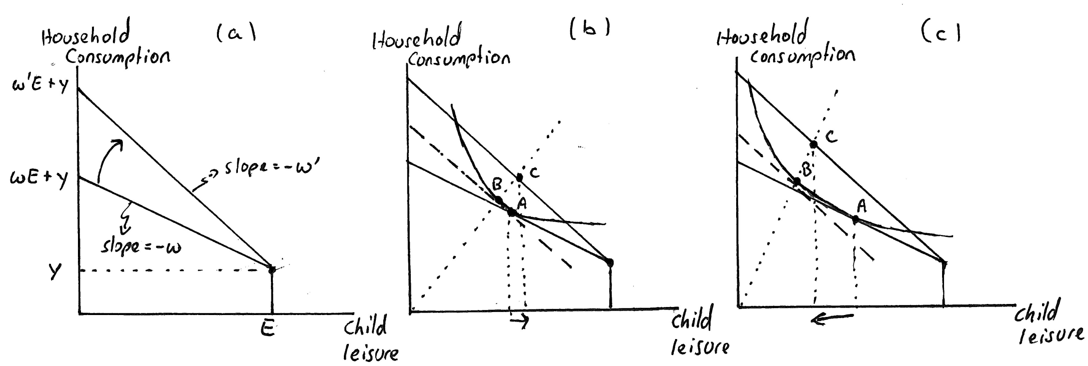
Exercise 8.9: *International Trade and Child Labor*

Policy Application: *International Trade and Child Labor: The economist Jagdish Bhagwati explained in one of his public lectures that international trade causes the wage for child labor to increase in developing countries. He then discussed informally that this might lead to more child labor if parents are "bad" and less child labor if parents are "good".*

A: *Suppose that households in developing countries value two goods: "Leisure time for Children in the Household" and "Household Consumption." Assume that the adults in a household are earning $y in weekly income regardless of how many hours their children work. Assume that child wages are w per hour and that the maximum leisure time for children in a household is E hours per week.*

(a) *On a graph with "weekly leisure time for children in the household" on the horizontal axis and "weekly household consumption" on the vertical, illustrate the budget constraint for a household and label the slopes and intercepts.*

Answer: This initial budget is illustrated in panel (a) of Graph 8.15 where the bundle (E, y) is effectively the "endowment" bundle for the household — i.e. the bundle that does not depend on child wages.



Graph 8.15: Child Labor and International Trade

(b) *Now suppose that international trade expands and, as a result, child wages increase to w'. Illustrate how this will change the household budget.*

Answer: This is also illustrated in panel (a) of the graph — the budget rotates outward around the "endowment" bundle (E, y).

(c) *Suppose that household tastes are homothetic and that households require their children to work during some but not all the time they have available. Can you tell whether children will be asked to work more or less as a result of the expansion of international trade?*

Answer: You cannot tell — it depends on the size of the substitution effect and thus on the degree of substitutability between child leisure and household consumption. We know that tastes can be homothetic with little or no substitutability between goods (as in perfect complements), and tastes can be homothetic with perfect substitutability. Of course there are lots of in between cases. In panel (b), we illustrate the case of relatively little substitutability where the substitution effect from A to B is small and outweighed by the wealth effect from B to C to result in an overall increase in leisure for children. In panel (c), on the other hand, we illustrate the case where the substitution effect outweighs the wealth effect — resulting in a decrease in leisure for children.

The substitution effect here simply says that, as child wages increase, the opportunity cost of giving leisure to children increases and households will therefore give less leisure. The wealth effect, on the other hand, says that increasing child wages make the household richer — and richer households will consume more of all normal goods, including child leisure.

(d) *In the context of the model with homothetic tastes, what distinguishes "good" parents from "bad" parents?*

Answer: Good parents are those whose tastes look more like those in panel (b) while bad parents are those whose tastes look more like panel (c). Put differently, parents become "better" in this model the more they view child leisure and household consumption as complements. This has a certain amount of intuitive appeal: Good parents are those that essentially say that they can only become better off when household consumption goes up if child welfare (i.e. child leisure) also goes up — they are complements and have to go together. Bad parents are those that view household consumption as a substitute for child welfare.

(e) *When international trade increases the wages of children, it is likely that it also increases the wages of other members of the household. Thus, in the context of our model, y — the amount brought to the household by others — would also be expected to go up. If this is so, will we observe more or less behavior that is consistent with what we have defined as "good" parent behavior?*

Answer: This would cause a parallel shift in the budget beyond the initial rotation that results from the increase child wages. Such a parallel shift gives rise to a pure wealth effect. So long as child leisure is a normal good, increases in y would therefore cause increases in consumption of all goods — including child leisure. This would strengthen the wealth effect from the increase in w and thus cause more parents to reduce the amount of work their children have to undertake. Put differently, the more y is also increased by international trade, the more substitutable child leisure and household consumption can be and still cause parents to be "good".

(f) *In some developing countries with high child labor rates, governments have instituted the following policy: If the parents agree to send a child to school instead of work, the government pays the family an amount x. (Assume the government can verify that the child is in fact sent to school and does in fact not work, and assume that the household views time at school as leisure time for the child.) How does that alter the choice set for parents? Is the policy more or less likely to succeed the more substitutable the household tastes treat child "leisure" and household consumption?*

Answer: Under this policy, the government in essence makes one additional bundle available to the household — a bundle in which the child's "leisure" or "non-work" hours are E and the household's consumption is y plus the payment x the government is providing in order for the child to attend school. This new bundle is depicted as bundle B in both panels of Graph 8.16. In each panel, A is the original optimal bundle before this policy was introduced. But in panel (a), the original optimal indifference curve is relatively flat and therefore passes below B while in panel (b) it is closer to the shape of perfect complements which makes it pass above B. Thus, conditional on A being the original optimum, the policy is more likely to induce the household to choose B (and thus send their child to school) the more substitutable are household consumption and child leisure.

B: *Suppose parental tastes can be captured by the utility function $u(c,\ell) = c^{0.5}\ell^{0.5}$. For simplicity, suppose further that $y = 0$.*

(a) *Specify the parents' constrained optimization problem and set up the appropriate Lagrange function.*

Answer: The problem to be solved is

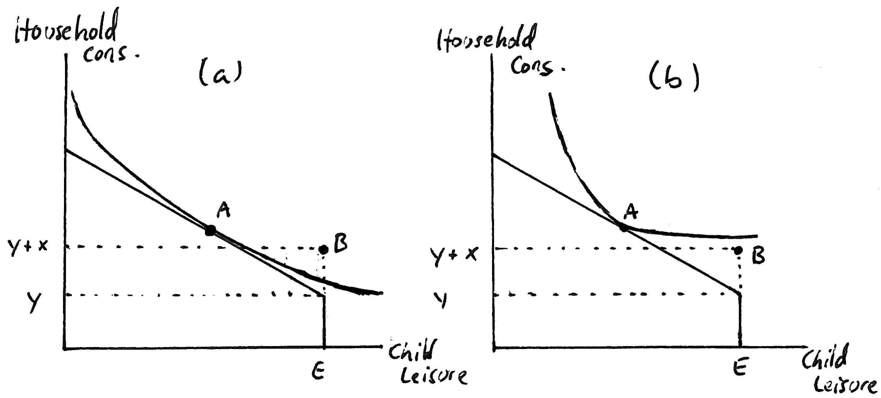
$$\max_{c,\ell} c^{0.5}\ell^{0.5} \text{ subject to } c = w(E-\ell). \tag{8.48}$$

The Lagrange function is

$$\mathcal{L}(c,\ell,\lambda) = c^{0.5}\ell^{0.5} + \lambda(w(E-\ell)-c). \tag{8.49}$$

(b) *Solve the problem you have set up to determine the level of leisure the parents will choose for their children. Does w have any impact on this decision?*

Solving the first two first order conditions, you get $c = w\ell$. Plugging this into the budget constraint, you can then solve for $\ell = E/2$ and plugging this back into $c = w\ell$, you can get $c = wE/2$. The level of leisure parents choose for their children ($\ell = E/2$) is independent of wage — so w has no impact on their decision in this case.



Graph 8.16: Child Labor and International Trade: Part II

(c) *Explain intuitively what you have just found. Consider the CES utility function (that has the Cobb-Douglas function you just worked with as a special case). For what ranges of ρ would you expect us to be able to call parents "good" in the way that Bhagwati informally defined the term?*

Answer: For the Cobb-Douglas tastes that are modeled, the substitution effect (that causes parents to reduce their children's leisure when w increases) is exactly offset by the wealth effect (which causes parents to increase their children's leisure as w increases). We know that Cobb-Douglas tastes are CES tastes with $\rho = 0$ and elasticity of substitution of 1. As ρ falls below zero, the goods become more substitutable and as ρ rises above zero they become more complementary. In part A we determined that parents are more likely to be "good" if they view child leisure as relatively complementary to household consumption — thus, for CES utility functions, parents are "bad" if $-1 \leq \rho < 0$ and parents are "good" if $0 < \rho \leq \infty$.

(d) *Can parents for whom household consumption is a quasilinear good ever be "good"?*

Anwer: Yes, if substitution effects are sufficiently small, such parents can be "good". This is because tastes that are quasilinear *in consumption* would only give rise to substitution effects with no wealth effect for household consumption (i.e. *on the vertical axis*). Thus, while the substitution effect points to an increase in household consumption and a decrease in child leisure, the wealth effect points to no further change in household consumption and an increase in child leisure. Put differently, while the quasilinearity of household consumption implies no wealth effect on the vertical axis, it also implies the entire wealth effect happens on the child leisure axis in the direction opposite to the substitution effect.

Be careful in this answer to pay attention to the fact that the question states that household consumption, not child leisure, is the quasilinear good. Had the question asked whether parents can be "good" if child leisure is the quasilinear good, the answer would have been an unambiguous no. This is because we would then only have a substitution effect on the horizontal axis — which implies that child leisure decreases and thus child labor increases with an increase in w.

(e) *Now suppose (with the original Cobb-Douglas tastes) that $y > 0$. If international trade pushes up the earnings of other household members — thus raising y, what happens to child leisure?*

Answer: Solving for the optimal leisure time (in the same way as we did above), we get

$$\ell = \frac{wE + y}{2w}. \tag{8.50}$$

The derivative of this with respect to y is positive — i.e. as y increases, so does the amount of leisure chosen for the child.

(f) *Suppose again that $y = 0$ and the government introduces the policy described in part A(f). How large does x have to be in order to cause our household to send their child to school (assuming again that the household views the child's time at school as leisure time for the child)?*

Answer: Without participating in the policy, the household consumes $c = wE/2$ and $\ell = E/2$ — and therefore gets utility $(wE/2)^{0.5}(E/2)^{0.5} = w^{0.5}E/2$. If the household participates in the policy, it's child would get leisure of E and the household consumption would be x. Thus, participating in the policy means utility of $x^{0.5}E^{0.5}$. The household will be indifferent between the two options if the utility of participating and not participating are equal; i.e. if

$$\frac{w^{0.5}E}{2} = x^{0.5}E^{0.5}. \tag{8.51}$$

Solving this for x we get $x = wE/4$. For any x greater than this, the household is therefore better off choosing to send their child to school.

(g) *Using your answer to the previous part, put into words what fraction of the market value of the child's time the government has to provide in x in order for the family to choose schooling over work for their child?*

Answer: We concluded above that x has to be at least $wE/4$ in order for the household to be willing to send the child to school. The market value of the child's time endowment is wE. The amount that is required for the child to be sent to school is therefore equivalent to one quarter of the market value of the leisure time of the child.

Conclusion: Potentially Helpful Reminders

1. If this chapter seems difficult, it is probably because you have not yet fully internalized Chapter 7. This is because the material of this chapter is conceptually almost identical to the material in the previous chapter.

2. Remember to never think about wealth effects unless you have two parallel budgets to work with. Also remember never to allow a substitution effect to move you off an indifference curve.

3. When applying definitions like normal and inferior goods, or definitions like homothetic or quasilinear tastes, always be sure you are doing so when developing the wealth effect that takes you from one budget to a parallel budget.

4. Try to make intuitive sense of substitution and wealth effects in each application. Substitution effects always point in the direction of more consumption of what's become cheaper and less consumption of what's become more expensive. Wealth effects in labor and capital markets almost always involve normal goods — and thus point in the direction of the wealth change.

5. When wealth and substitution effects point in opposite directions, your answer will typically be ambiguous: If the substitution effect is small because the goods are fairly complementary, the wealth effect will dominate; but if the substitution effect is large because the goods are fairly substitutable, then the substitution effect will dominate.

6. Homothetic tastes, for instance, can have small or large substitution effects depending on whether the indifference curves are relatively L-shaped or relatively flat. End-of-chapter exercise 8.1 illustrates this, and exercise 8.9 develops the idea in an intriguing application.