CHAPTER

9

Demand for Goods and Supply of Labor and Capital

Now we have finally arrived at the point where we talk about demand curves. These summarize the relationship between some aspect of the consumer's economic circumstances and the quantity she demands of a particular good. Although we typically think of demand curves as illustrating the relationship between quantity of x demanded and the price of x, demand curves can also illustrate the relationship between quantity and income or between quantity and the price of some other good. Each of these demand curves is just a "slice" of a more complicated demand relationship (that we call a demand function in part B) — with that slice holding all but one of the economic variables that define a consumer's economic circumstances fixed. And, just as demand curves emerge from the consumer's diagram, supply curves for labor and capital emerge from the worker's and saver's diagrams (and demand curves for capital emerge from the borrower's diagram.)

Chapter Highlights

The main points of the chapter are:

1. **Demand and supply relationships illustrate how choices depend on aspects of the economic environment** — where the economic environment potentially includes income and prices for different types of goods.

2. When we isolate the impact of one particular aspect of that economic environment, we are implicitly holding all other aspects of that environment fixed — i.e. **we are graphing a "slice" of a more complicated function** that tells us how behavior changes as all these aspects of our economic environment change.

3. While we can illustrate **these relationships** as demand and supply curves, they ultimately **emerge from the underlying choice model** we have developed and can be understood only with that framework in mind.

4. Income-demand relationships depend only on income effects — and thus the relationship of indifference curves to one another. **Price-demand relationships depend on both income and substitution effects** and thus also depend on the degree of substitutability between different goods.

5. **Labor and capital supply relationship** often involve wealth and substitution effects that point in opposite direction. This implies that, despite all goods (typically) being normal goods, the labor and capital supply curves can **slope up or down depending on which effect dominates**.

Using the LiveGraphs

For an overview of what is contained on the LiveGraphs site for each of the chapters (from Chapter 2 through 29) and how you might utilize this resource, see pages 2-3 of Chapter 1 of this *Study Guide*. To access the LiveGraphs for Chapter 9, click the *Chapter 9* tab on the left side of the LiveGraphs web site.

In addition to the *Animated Graphics*, *Static Graphics* and *Downloads* portions of the LiveGraphs site, this Chapter has one **Exploring Relationships** module that goes in more detail into the different possible ways that own-price demand curves can look depending on what type of good we are modeling. It's a nice summary of what you find in some of the *Animated Graphics*.

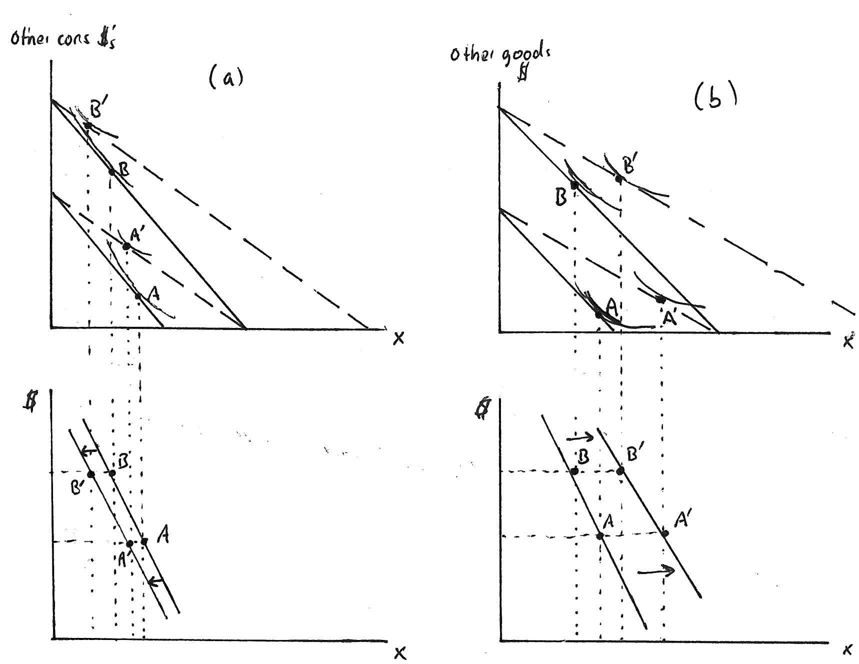
9A Solutions to Within-Chapter Exercises for Part A

Exercise 9A.1 *In an earlier chapter, we mentioned that it is not possible for a good to be inferior for all income levels. Can you see in the lower panel of Graph 9.1a why this is true?*

Answer: In order for pasta to be inferior for all income levels, the relationship between pasta on the horizontal axis and income on the vertical would have to continue to slope downward as income falls — eventually intersecting the pasta axis. But without any income, the consumer would in fact not be able to buy any pasta.

Exercise 9A.2 *Suppose good x is an inferior good for an individual. Derive the income-demand curve as in Graph 9.1a. Then graph a decrease in the price for x for both income levels in the top panel — and show how this affects the income-demand curve in the lower panel depending on whether x is Giffen or regular inferior.*

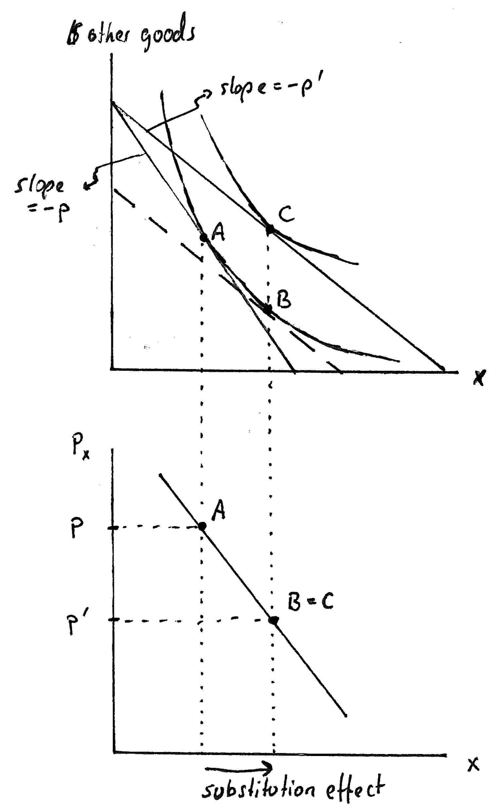
Answer: In the top portions of panels (a) and (b) of Graph 9.1, two initial (solid) budgets are drawn representing two income levels at some initial price. From these, points A and B are derived just as in the text. Then, two new (dashed) budgets are added — these differ from the original budgets only in that the price of x has fallen. In panel (a), the new optimal bundles A' and B' are to the left of A and B (respectively) — indicating that a decrease in the price causes a decrease in consumption of x. Thus, in panel (a), x is a Giffen good. When translated to the lower graph, A' and B' now appear to the left of A and B (respectively) — thus, the decrease in the price of x causes a leftward shift in the income-demand graph. In panel (b), on the other hand, a decrease in price causes the new optima A' and B' to lie to the right of A and B (respectively) — indicating that a decrease in the price of x causes an increase in the consumption of x. Thus, panel (b) represents the case when x is not Giffen (and is regular inferior). When translated to the lower graph, A' and B' now lie to the right of A and B (respectively) — i.e. a decrease in the price of x causes a rightward shift of the income-demand curve. (*Note*: The vertical axis in the lower graphs is labeled as dollars — but it is actually measuring *income*. You could — and perhaps should — therefore label the axis as *Income in Dollars*.)



Graph 9.1: Shifts in Income-Demand Curves

Exercise 9A.3 *Repeat the derivation of own-price demand curves for the case of quasilinear tastes and explain in this context again how quasilinear tastes are borderline tastes between normal and inferior goods.*

Answer: This is derived in Graph 9.2. In the case of quasilinear goods, there is no income effect from a price change (for that good) — as a result, the shape of the demand curve arises entirely from the substitution effect (with *B* and *C* coinciding on the lower graph). In the cases treated in the text, we showed that *B* lies to the left of *C* for normal goods and to the right of *C* for inferior (both regular and Giffen) goods. The quasilinear case is the borderline case where *B* lies on top of *C* due to the absence of an income effect. Put differently, the graphs in the text show that the income effect points in the same direction as the substitution effect for normal goods and in the opposite direction for inferior goods. For quasilinear goods, it is simply absent.



Graph 9.2: Demand Curve for Quasilinear Case

Exercise 9A.4 *How would the own price demand curves in Graphs 9.2a through (c) change with a decrease in income? (*Hint*: Your answer for panel (a) should be different than your answers for panels (b) and (c).)*

Answer: Consider first the case of x being a normal good. A normal good is one where, if income decreases, the consumer consumes less (all else being the same). In panel (a) (of the graph in the text), you can already see what happens at the lower price if income falls — B is the optimal bundle at a lower income level and C is optimal at a higher income level, with prices (i.e. the slope of the budget) the same in both cases. Thus, if income falls by the difference between these budgets, the new demand curve will go through B. The same must be true at every other price level — if income falls (and price stays at that price level), consumption falls. Thus, the entire demand curve shifts to the left with a decrease in income.

In panels (b) and (c), on the other hand, the opposite is the case. As income falls from the higher to the lower of the two parallel budgets, consumption *increases*. The new demand curve will again go through B in the lower panel, but now B lies to the right of the original demand curve. Similarly, for any other price level, if income falls, the consumer will consume more if x is inferior — thus the new demand curve lies to the right of the original.

Exercise 9A.5 *What kind of good would x have to be in order for the demand curve not to shift as income changes?*

Answer: The good x would have to be quasilinear — i.e. in the absence of income effects, changes in income do not change how much a consumer will buy for a given price.

Exercise 9A.6 *What kind of good would x_1 have to be in order for this cross-price demand curve to slope down?*

Answer: For the cross-price demand curve to slope down, point C in the graph in the text would have to lie to the right of A. Since the substitution effect pushes us in the direction of *less* x_1 as p_2 falls, the income effect would have to push us in the other direction (and outweigh the substitution effect). Thus, the income effect would have to be positive — implying that x_1 has to be a normal good.

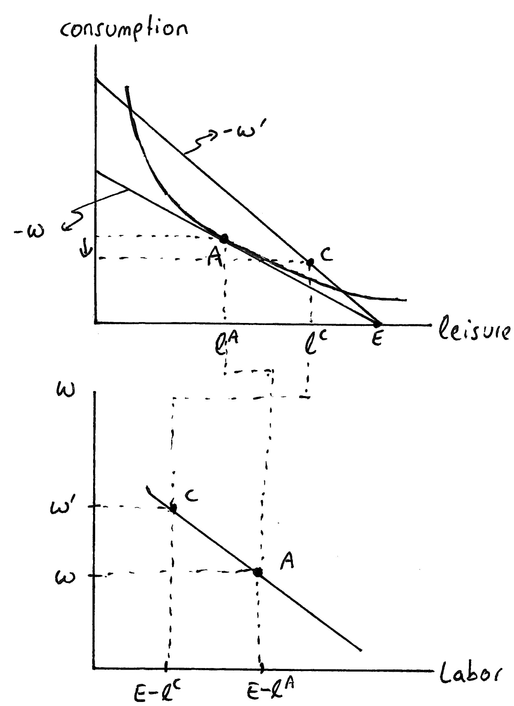
Exercise 9A.7 *In our analysis of consumer goods, we usually found that income and substitution effects point in the same direction when goods are normal. Why are wealth and substitution effects now pointing in opposite directions when leisure is a normal good?*

Answer: The wage is the opportunity cost of consuming leisure. But the budget in this case is not exogenous (as it was in the case of goods in the previous section). Rather, the budget arises from the fact that we *own* our leisure endowment. When the wage goes up, our leisure endowment becomes more valuable — so we have in essence become richer. This is in contrast to the price of goods in the previous section going up — there an increase in a price made us poorer. The wealth effect therefore now points in the opposite direction because the "price" increase (i.e. the wage increase) increases rather than decreases our "income".

You can also view this in strictly mechanical terms. A change in the wage rate does not change the leisure-intercept of our budget. Rather, it changes the consumption intercept. So, in strictly technical terms, an increase in the wage actually looks like a decrease in the price of consumption — the good on the vertical axis. This is analogous to what we graphed in the textbook in Graph 9.3 where we showed a cross-price demand curve where income and substitution effects operate exactly as they do in the leisure/consumption graph when wages change.

Exercise 9A.8 True or False*: Leisure being an inferior good is* sufficient *but not* necessary *for labor supply to slope up.*

Answer: This is true. In panel (c) of Graph 9.4 in the text, we show that when leisure is inferior, the labor supply curve must slope up. So leisure being inferior is sufficient for an upward slope of labor supply. But we also show in panel (b) that labor supply *can* slope up when leisure is normal. So it is not necessary for leisure to be inferior in order for labor supply to slope up.



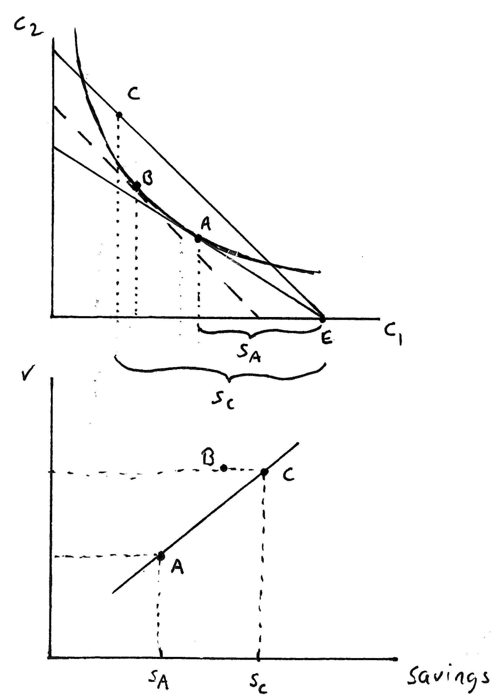
Graph 9.3: Labor Supply when Consumption Giffen

Exercise 9A.9 *Can you tell which way the labor supply curve will slope in the unlikely event that "other consumption" is a Giffen good?*

Answer: Yes — the labor supply will slope down. This is graphed in Graph 9.3 (on the previous page) where C in the top panel must lie *below* A. This is because an increase in the wage when Leisure is an endowment is technically equivalent to a decrease in the price of consumption when the budget is treated exogenously — and consumption is Giffen if such a decrease in its price causes less consumption. But if C lies below A, it must also lie to the right of A — i.e. the increase in the wage causes an increase in leisure consumption — and thus a decrease in work.

Exercise 9A.10 *Would the interest rate/savings curve slope up or down if consumption this period were an inferior good?*

Answer: The substitution and wealth effects would now point in the same direction for current consumption — which means that the curve would slope up. This is depicted in Graph 9.4.



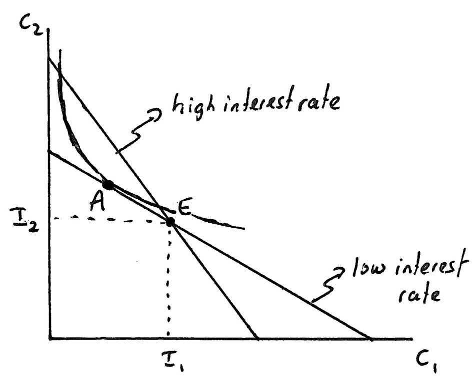
Graph 9.4: Consumption this Period as Inferior Good

Exercise 9A.11 *What kind of good would consumption this summer have to be in order for the interest rate/borrowing relationship to be positive in Graph 9.6?*

Answer: Consumption this summer would have to be Giffen. You can see this most easily by just graphing the initial and final budget (without the compensated budget). In this graph, an increase in the interest rate is technically equivalent to an increase in the price of current consumption when income is exogenously modeled. In order for the borrowing curve to slope up, C would have to lie to the right of A — i.e. as the price of current consumption goes up, you would have to do more of it. That's the definition of a Giffen good.

Exercise 9A.12 *Is it possible for someone to begin as a saver at low interest rates and switch to become a borrower as the interest rate rises?*

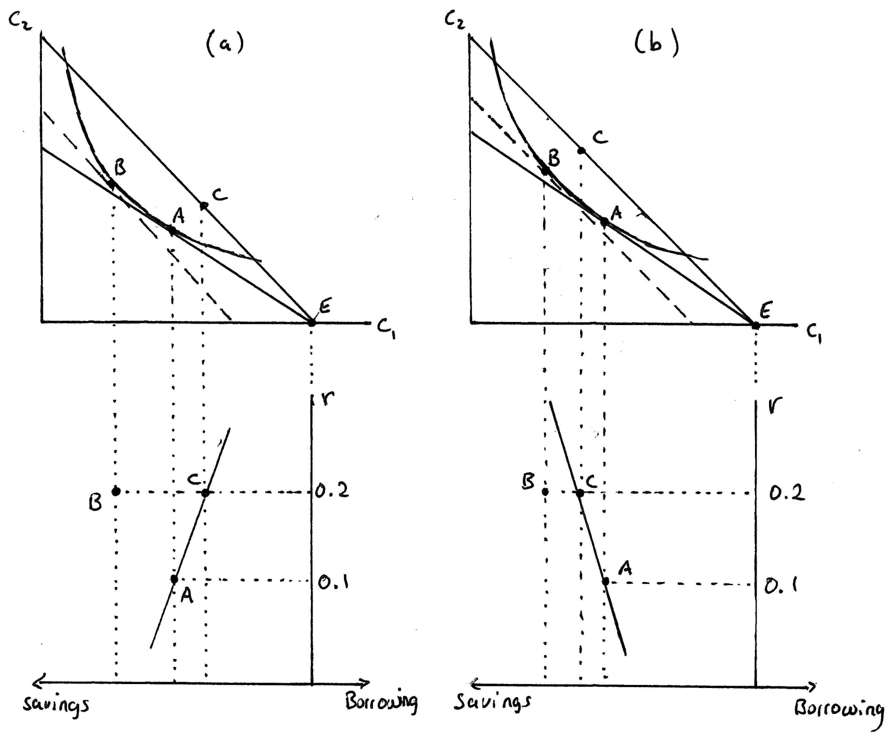
Answer: In Graph 9.5 we illustrate a high and low interest rate budget constraint for someone with income both now and in the future (so that it in principle it is possible for him to be either a borrower or a saver). If he is a saver under the low interest rate, his optimum looks something like bundle A (to the left of E). But if an indifference curve is tangent at A, it must necessarily lie above E — which implies no bundle to the left of E under either budget could also be optimal. Thus, it is not possible for an increase in the interest rate to induce a saver to become a borrower.



Graph 9.5: Not possible to switch from Saver to Borrower as r Increases

Exercise 9A.13 *The technique of placing the axis below the endowment point E developed in Graph 9.7 could also be applied to the previous two graphs, Graph 9.5 and Graph 9.6. How would those graphs change?*

Answer: This is illustrated in Graph 9.6 (next page) for the case where all income is in this period and no income is expected in the next period (as in Graph 9.5 in the textbook). For the textbook Graph 9.6, nothing would change because the axis would be placed exactly as it is in the graphs in the text since the "endowment" point occurs at zero current income.

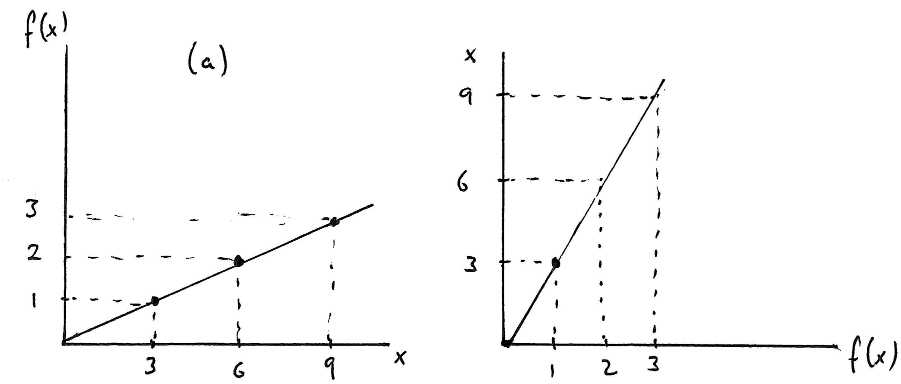


Graph 9.6: Savings as in Textbook Graph 9.5

9B Solutions to Within-Chapter Exercises for Part B

Exercise 9B.1 *Consider the function $f(x) = x/3$. Graph this as you usually would with x on the horizontal axis and $f(x)$ on the vertical. Then graph the inverse of the function, with $f(x)$ on the horizontal and x on the vertical.*

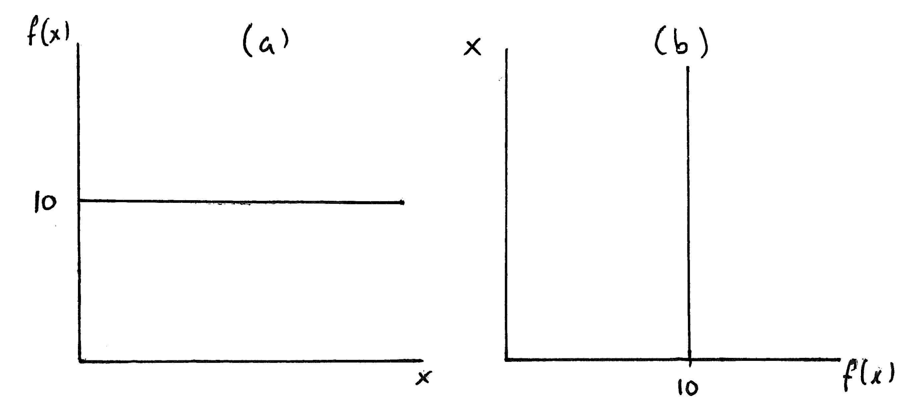
Answer: This is done in panels (a) and (b) of Graph 9.7 (on the next page).



Graph 9.7: Direct and Inverse Graph of $f(x) = x/3$

Exercise 9B.2 *Repeat the previous exercise for the function* $f(x) = 10$.

Answer: This is done in panels (a) and (b) of Graph 9.8.



Graph 9.8: Direct and Inverse Graph of $f(x) = 10$

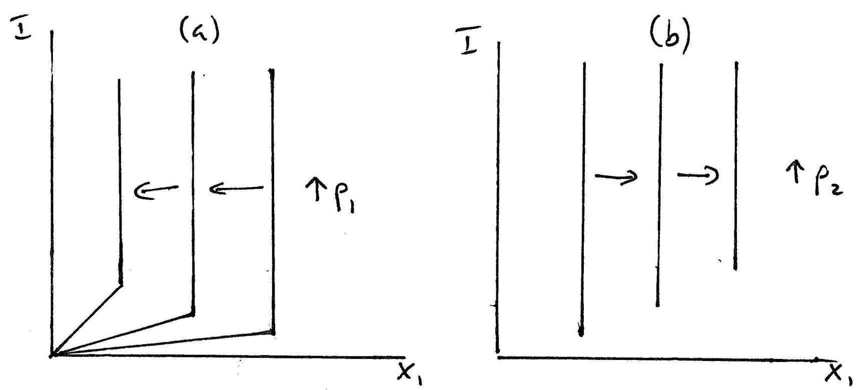
Exercise 9B.3 *Another special case of tastes that we have emphasized throughout is the case of quasilinear tastes. Consider, for instance, the utility function $u(x_1, x_2) = 100(\ln x_1) + x_2$. Calculate the demand function for x_1 and derive some sample income–demand curves for different prices.*

Answer: The Lagrange function is

$$\mathcal{L}(x_1, x_2, \lambda) = 100(\ln x_1) + x_2 + \lambda(I - p_1 x_1 - p_2 x_2). \tag{9.1}$$

The first two first order conditions solve to $x_1 = 100p_2/p_1$. (Plugging this back into the budget constraint, we can also solve for $x_2 = (I - 100p_2)/p_2$.)

In panel (a) of Graph 9.9, we graph these as p_1 changes. Note that I does not enter the demand functions for x_1 — so changes in income do not alter consumption of x_1. This should not be surprising — quasilinear goods are goods for which there are no income effects. Technically, at some point income does, however, enter — when income falls too low, we cannot afford what the demand function tells us and we end up at a corner solution. As p_1 increases, that corner solution begins to happen at lower income levels. Panel (b) illustrates changes as p_2 increases — with the portion that occurs when there is a corner solution not graphed.



Graph 9.9: Income Demand Graphs for Quasilinear good

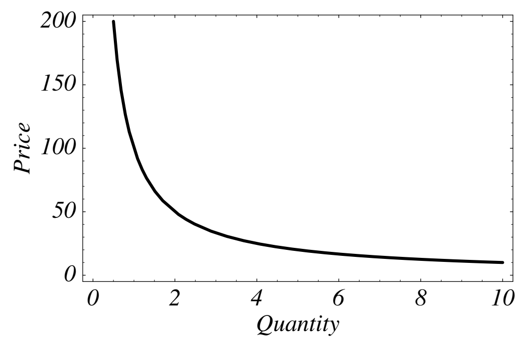
Exercise 9B.4 *Can you derive the same result for x_2?*

Answer: We would get

$$\frac{\partial p_2}{\partial x_2} = -\frac{(1-\alpha)I}{x_2^2} = -\frac{(1-\alpha)I}{((1-\alpha)I/p_2)^2} = -\frac{p_2^2}{(1-\alpha)I} = \left(\frac{\partial x_2}{\partial p_2}\right)^{-1}. \tag{9.2}$$

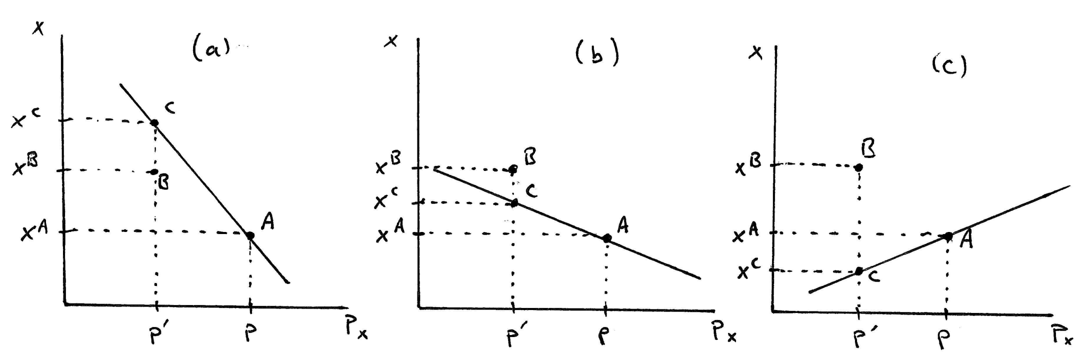
Exercise 9B.5 *As in exercise 9B.3, consider again tastes that can be represented by the utility function $u(x_1, x_2) = 100(\ln x_1) + x_2$. Using the demand function for x_1 that you derived in the previous exercise, plot the own-price demand curve when income is 100 and when $p_2 = 1$. Then plot the demand curve again when income rises to 200. Keep in mind that you are actually plotting inverse functions as you are doing this.*

Answer: The demand function we derived previously is $x_1 = 100p_2/p_1$. When $p_2 = 1$, this becomes $x_1 = 100/p_1$. The inverse is $p_1 = 100/x_1$, which is plotted in Graph 9.10. Since I does not enter the function, changes in income do not shift it.



Graph 9.10: Own-Price Demand Curve for Quasilinear good

Exercise 9B.6 *Knowing that own price demand curves are inverse slices of own price demand functions, how would the lower panels of Graph 9.2 look if you graphed slices of the actual functions (rather than the inverses) — i.e. when you put price on the horizontal and the quantities of goods on the vertical axis?*



Graph 9.11: Direct Demand Curves Corresponding to Graph 9.2 in the Text

Exercise 9B.7 *What would the slices of the demand function (rather than the inverse slices in Graph 9.10a) look like?*

Answer: They would simply be horizontal rather than vertical lines.

Exercise 9B.8 *Verify that these are in fact the right demand functions for tastes represented by the CES utility function.*

Answer: The Lagrange function for the maximization problem is

$$\mathscr{L}(x_1, x_2, \lambda) = (\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho})^{-1/\rho} + \lambda(I - p_1 x_1 - p_2 x_2). \tag{9.3}$$

The first two first order conditions are then

$$\begin{aligned} \frac{\partial \mathscr{L}}{\partial x_1} &= -\frac{1}{\rho}\left(\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho}\right)^{-1/\rho}\left(-\rho\alpha x_1^{-\rho-1}\right) - \lambda p_1 = 0 \\ \frac{\partial \mathscr{L}}{\partial x_2} &= -\frac{1}{\rho}\left(\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho}\right)^{-1/\rho}\left(-\rho(1-\alpha) x_2^{-\rho-1}\right) - \lambda p_2 = 0 \end{aligned} \tag{9.4}$$

which can also be written as

$$\begin{aligned} \left(\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho}\right)^{-1/\rho} \alpha x_1^{-\rho-1} &= \lambda p_1 \\ \left(\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho}\right)^{-1/\rho} (1-\alpha) x_2^{-\rho-1} &= \lambda p_2. \end{aligned} \tag{9.5}$$

Dividing these two equations by one another, we get (after canceling terms)

$$\frac{\alpha}{(1-\alpha)}\left(\frac{x_2}{x_1}\right)^{\rho+1} = \frac{p_1}{p_2} \tag{9.6}$$

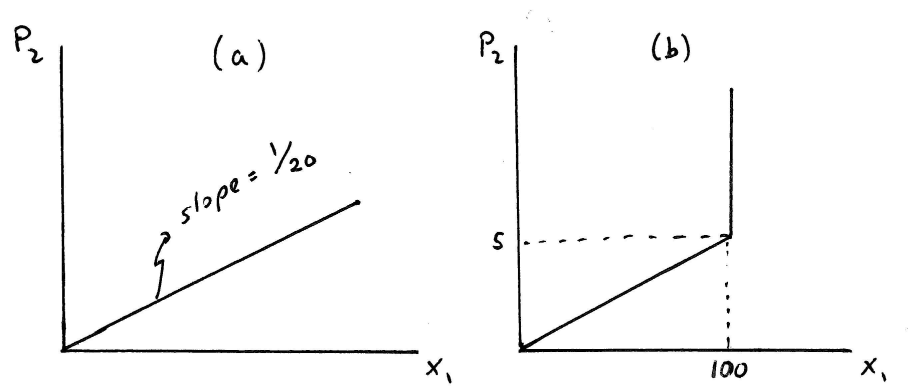
or

$$x_2 = \left(\frac{(1-\alpha)p_1}{\alpha p_2}\right)^{1/(\rho+1)} x_1. \tag{9.7}$$

Plugging this into the budget constraint and solving for x_1 then gives us the demand equation in the text. Plugging that back into equation (9.7), we can also solve for x_2 — the demand equation given in the text.

Exercise 9B.9 *In Graph 9.3, we intuitively concluded that cross-price demand curves slope up when tastes are quasilinear. Verify this for tastes that can be represented by the utility function* $u(x_1, x_2) = 100(\ln x_1) + x_2$ *for which you derived the demand functions in exercise 9B.3. Draw the cross-price demand curve for* x_1 *when income is 2,000 and* $p_1 = 5$.

Answer: We determined previously that $x_1 = 100p_2/p_1$ is the demand function for x_1 in this case. The derivative of x_1 with respect to p_2 is then just $100/p_1$ which is greater than zero. Thus, the cross-price demand curve slopes up. When $p_1 = 5$, the demand function becomes simply $x_1 = 100p_2/5 = 20p_2$, and the inverse function becomes $p_2 = x_1/20$. This is graphed in panel (a) of Graph 9.12 (next page).



Graph 9.12: Cross-Price Demand Curve for Quasilinear tastes

Exercise 9B.10 *Suppose that income was 500 instead of 2,000 in exercise 9B.9. Determine at what point the optimization problem results in a corner solution (by calculating the demand function for x_2 and seeing when it becomes negative). Illustrate how this would change the cross price demand curve you drew in exercise 9B.9. (Hint: The change occurs in the cross price demand curve at $p_2 = 5$.)*

Answer: This is illustrated in panel (b) of Graph 9.12. In this example, you only have \$500 to spend. Your demand for x_1 is given by $x_1 = 100p_2/p_1$ which becomes $x_1 = 20p_2$ when $p_1 = 5$. Thus, when $p_2 = 5$, the demand function tells us you will buy $x_1 = 20(5) = 100$ — and at a price of $p_1 = 5$, this costs \$500 — your entire income. You therefore reach a corner solution where you buy no more of x_2 when p_2 reaches 5. For higher levels of p_2, you would then still be at the same corner solution — buying 100 units of x_1. (For the case where $I = 2,000$ as in the previous exercise, this corner solution is not reached until $p_2 = 20$ — which is where the kink would occur in panel (a) of the graph).

Exercise 9B.11 *What function is graphed in the middle portions of each panel of Graph 9.4? What function is graphed in the bottom portion of each panel of Graph 9.4?*

Answer: The function graphed in the middle panel is the inverse of the leisure demand function — or what we denoted $\ell(w,L)$ — with L held fixed. The function graphed in the lower panel is the inverse labor supply function — or what we denoted $l(w,L)$ — with L held fixed.

Exercise 9B.12 *Verify these results.*

Answer: The Lagrange function for this problem is

$$\mathcal{L}(c,\ell,\lambda) = c + \alpha \ln \ell + \lambda(w(L-\ell) - c). \tag{9.8}$$

The first two first order conditions are

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial c} &= 1-\lambda = 0\\ \frac{\partial \mathscr{L}}{\partial \ell} &= \frac{\alpha}{\ell} - \lambda w = 0.\end{aligned} \tag{9.9}$$

Solving these for ℓ we get $\ell = \alpha/w$. Plugging this back into the budget constraint $c = w(L-\ell)$, we then also get $c = wL-\alpha$. Finally, the labor supply function is simple $L-\ell$ — or $l = L-\alpha/w$.

Exercise 9B.13 *Verify this leisure demand and labor supply function for the CES function that is given.*

Answer: The Lagrange function for this problem is

$$\mathscr{L}(c,\ell,\lambda) = \left(\alpha c^{-\rho} + \beta \ell^{-\rho}\right)^{-1/\rho} + \lambda(w(L-\ell)-c). \tag{9.10}$$

The first two first order conditions quickly solve to (after canceling a number of items) to

$$\frac{\alpha}{\beta}\left(\frac{\ell}{c}\right)^{\rho+1} = \frac{1}{w} \tag{9.11}$$

or

$$c = \left(\frac{\alpha w}{\beta}\right)^{1/(\rho+1)} \ell. \tag{9.12}$$

Substituting this into the budget constraint $c = w(L-\ell)$ and solving for ℓ, we get the leisure demand equation in the text. Subtracting this from L then gives us the labor supply equation.

Exercise 9B.14 *Verify that these three equations are correct.*

Answer: The Lagrange function for this problem is

$$\mathscr{L}(c,\ell,\lambda) = c^{\alpha}\ell^{(1-\alpha)} + \lambda((1+r)e_1 + e_2 - (1+r)c_1 - c_2). \tag{9.13}$$

The first two first order conditions solve to give us

$$c_2 = \frac{(1+r)(1-\alpha)c_1}{\alpha}. \tag{9.14}$$

Plugging this into the budget constraint $(1+r)e_1 + e_2 = (1+r)c_1 + c_2$ and solving for c_1, we get the expression $c_1(r,e_1,e_2)$ in the text. Substituting this back into equation (9.14) and solving for c_2 then gives $c_2(r,e_1,e_2)$ in the text. Finally, $s(r,e_1,e_2)$ is just $c_1(r,e_1,e_2)$ subtracted from period 1 endowment e_1.

Exercise 9B.15 *Consider the more general CES utility function $u(c_1, c_2) = (0.5c_1^{-\rho} + 0.5c_2^{-\rho})^{-1/\rho}$ and solve for the savings supply function when you earn \$10,000 this period and nothing in the future. Then verify that you obtain the vertical relationship between savings and the interest rate when $\rho = 0$ and determine how this slope changes when $\rho > 0$ (implying relatively low elasticity of substitution) and when $\rho < 0$ (implying relatively high elasticity of substitution).*

Answer: The Lagrange function is

$$\mathscr{L}(c_1, c_2, \lambda) = \left(0.5c_1^{-\rho} + 0.5c_2^{-\rho}\right)^{-1/\rho} + \lambda(10000(1+r) - (1+r)c_1 - c_2). \tag{9.15}$$

The first two first order conditions give (after some canceling of terms)

$$\left(\frac{c_2}{c_1}\right) = (1+r)^{1/(\rho+1)} \tag{9.16}$$

or

$$c_2 = (1+r)^{1/(\rho+1)} c_1. \tag{9.17}$$

Substituting this into the budget constraint $10000(1+r) = (1+r)c_1 - c_2$ and solving for c_1, we get

$$c_1 = \frac{10,000}{1 + (1+r)^{-\rho/(\rho+1)}} \tag{9.18}$$

and a savings supply function of

$$s = 10,000 - \frac{10,000}{1 + (1+r)^{-\rho/(\rho+1)}}. \tag{9.19}$$

To determine the relationship between the interest rate r and savings s, all we need to do is take the derivative of s with respect to r; i.e.

$$\frac{\partial s}{\partial r} = \left(\frac{-\rho}{\rho+1}\right)\left[\frac{10,000}{\left(1 + (1+r)^{\rho/(\rho+1)}\right)^2}(1+r)^{\frac{-2\rho-1}{\rho+1}}\right]. \tag{9.20}$$

Note that the term in large brackets is positive. Thus, $\partial s/\partial r$ is greater than or less than zero depending on whether $-\rho/(\rho+1)$ is greater than or less than zero. Thus,

$$\begin{aligned} &\frac{\partial s}{\partial r} < 0 \text{ if and only if } \rho > 0 \\ &\frac{\partial s}{\partial r} = 0 \text{ if and only if } \rho = 0 \\ &\frac{\partial s}{\partial r} > 0 \text{ if and only if } \rho < 0. \end{aligned} \tag{9.21}$$

Exercise 9B.16 *Using the CES utility function from exercise 9B.15, verify that the negative relationship between borrowing and the interest rate arises regardless of the value that ρ takes (whenever $e_1 = 0$ and $e_2 > 0$.)*

Answer: Solving a slight variant of the problem we solved in the previous exercise, we set up a Lagrange function

$$\mathcal{L}(c_1, c_2, \lambda) = \left(0.5c_1^{-\rho} + 0.5c_2^{-\rho}\right)^{-1/\rho} + \lambda(e_2 - (1+r)c_1 - c_2). \tag{9.22}$$

The first order conditions (just as in the previous problem) imply

$$c_2 = (1+r)^{1/(\rho+1)} c_1. \tag{9.23}$$

Plugging this into the budget constraint — which now is $e_2 = (1+r)c_1 + c_2$, we can solve for

$$c_1 = \frac{e_2}{1 + r + (1+r)^{1/(\rho+1)}}. \tag{9.24}$$

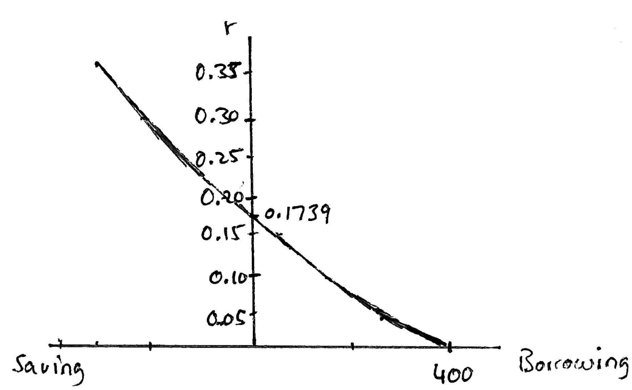
Since we are assuming that current income e_1 is zero, any consumption in the current period must come from borrowing. Thus, c_1 as just derived gives us the amount that the consumer chooses to borrow. The derivative of c_1 with respect to the interest rate r then tells us the relationship between borrowing and r. This derivative is

$$\frac{\partial c_1}{\partial r} = -\left[\frac{e_2}{1 + r + (1+r)^{1/(\rho+1)}}\left(1 + \frac{1}{\rho+1}(1+r)^{-\rho/(\rho+1)}\right)\right] \tag{9.25}$$

which is negative regardless of ρ (because the bracketed part is always positive but is preceded by a negative sign.)

Exercise 9B.17 *Graph this function in a graph similar to Graph 9.7 (which is the graph of an inverse borrowing (rather than saving) function).*

Answer: This is graphed in Graph 9.13.



Graph 9.13: Inverse Borrowing Function

End of Chapter Exercises

Exercise 9.2

The following is intended to explore what kinds of own-price demand relationships are logically possible in a two-good model with exogenous income (unless otherwise specified).

A: *For each of the following, indicate whether the relationship is possible or not and explain:*

(a) *Tastes are homothetic and the own-price demand relationship is positive.*

Answer: This is not possible. When tastes are homothetic, all goods are normal goods. Price increases for normal goods result in a negative substitution effect and a negative income effect in the same direction. Thus, the own-price demand relationship must be negative.

(b) *A good is inferior and its own-price relationship is negative.*

Answer: This is possible. When a good is inferior, then an increase in the price results in a negative substitution effect and a positive income effect. If the income effect is smaller than the substitution effect, the good is "regular inferior" — and the own-price demand curve is downward sloping (i.e. the own-price demand relationship is negative). (If the income effect is larger than the substitution effect, the own-price demand curve slopes up and the good is a Giffen good.)

(c) *In a model with endogenous income, a good is normal and its own-price demand relationship is negative.*

Answer: Yes, this is possible. Consider the case where all income is endogenously derived from owning a quantity E of x_1. This is illustrated in panel (a) of Graph 9.14 where the shallower budget corresponds to a lower price for good x_1 and the steeper budget to a higher price for x_1. In panel (b), the substitution effect from A to B is illustrated — clearly suggesting that, as p_1 increases, consumption of x_1 declines. If x_1 is a normal good (as specified in the question), the wealth effect then points in the opposite direction. It is therefore possible that the final optimal bundle at the higher price is C_1 — which results in a negative own-price demand relationship.

(d) *In a model with endogenous income, a good is normal and its own-price demand relationship is positive.*

Answer: This is also possible — as illustrated in panel (b) of Graph 9.14 (next page). The wealth effect could be large enough to make C_2 optimal after the price increase — in which case the own-price demand curve slopes up.

B: *Suppose that tastes can be represented by the Cobb-Douglas utility function* $u(x_1, x_2) = x_1^{\alpha} x_2^{(1-\alpha)}$.

(a) *Derive the demand functions when income is exogenous and illustrate that own-price demand curves slope down.*

Answer: The demand functions are

$$x_1 = \frac{\alpha I}{p_1} \text{ and } x_2 = \frac{(1-\alpha)I}{p_2}. \tag{9.26}$$

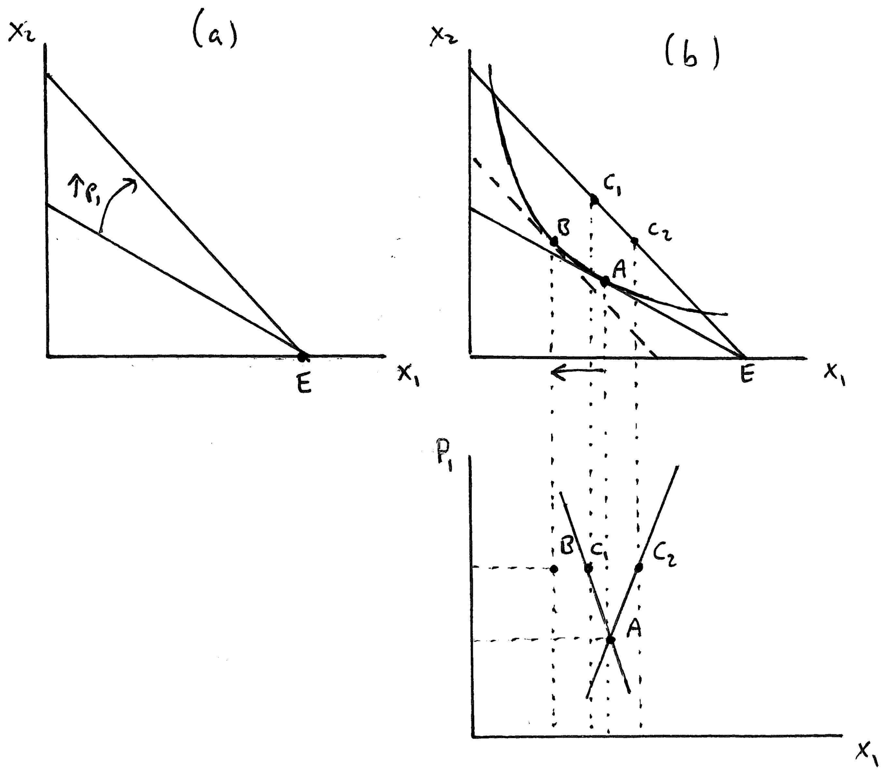
The derivatives of these with respect to own-price are negative — thus the demand curves slope down. (Technically, we'd want to take the derivatives of the inverse demand functions with respect to the goods in order to determine the slopes of the demand curves — but the sign of slopes does not change when we invert. Thus, we can simply take the derivative of the demand functions with respect to price to determine whether curves slope up or down.)

(b) *Now suppose that all income is derived from an endowment* (e_1, e_2). *If* $e_2 = 0$, *what is the shape of the own price demand curve for* x_1*?*

Answer: Solving the problem

$$\max_{x_1, x_2} x_1^{\alpha} x_2^{(1-\alpha)} \text{ subject to } p_1 x_1 + p_2 x_2 = p_1 e_1 + p_2 e_2 \tag{9.27}$$

in the usual way we get



Graph 9.14: Own-Price Demand Curves with Endogenous Incomes

$$x_1 = \frac{\alpha(p_1 e_1 + p_2 e_2)}{p_1} = \alpha e_1 + \frac{p_2}{p_1} e_2. \tag{9.28}$$

If $e_2 = 0$, this simply reduces to $x_1 = \alpha e_1$ — i.e. demand for x_1 does not depend on price. Thus, the demand curve is perfectly vertical at quantity $x_1 = \alpha e_1$.

(c) *Continuing with part (b), what is the shape of the own price demand curve for x_1 when $e_2 > 0$?*

<u>Answer</u>: When $e_2 > 0$, the derivative of x_1 with respect to p_1 is

$$\frac{\partial x_1}{\partial p_1} = -\frac{p_2}{p_1^2} e_2 < 0. \tag{9.29}$$

Thus, the demand curve slopes down when $e_2 > 0$.

(d) *Suppose tastes were instead represented by the more general CES utility function. Without doing any additional math, can you guess what would have to be true about ρ in order for the own-price demand for x_1 to slope up when $e_1 > 0$ and $e_2 = 0$?*

<u>Answer</u>: In Graph 9.14 of part A of this question, we illustrated that the own-price demand curve (for x_1) may slope up or down when all income is derived from and endowment of x_1 depending on whether the substitution effect is overcome by the positive wealth effect from a price increase. Thus, the smaller the substitution effect, the more likely it is that the

own-price demand curve slopes up. We also just showed that the own-price demand curve in this case is perfectly vertical when tastes are Cobb-Douglas — which is equivalent to the CES case when $\rho = 0$. Thus, $\rho = 0$ is the borderline case where the own-price demand curve neither slopes up nor down when income is endogenously derived from an endowment of x_1. It will slope up if there is less substitutability — and down if there is more. We know that the goods become less substitutable as ρ increases in the CES utility function — thus the own-price demand curve will slope up in our scenario when $\rho > 0$.

Exercise 9.5: *Backward-Bending Labor Supply Curve*

Everyday Application: *Backward-Bending Labor Supply Curve: We have suggested in this chapter that labor economists believe that labor supply curves typically slope up when wages are low and down when wages are high. This is sometimes referred to as a* backward bending labor supply curve.

A: *Which of the following statements is inconsistent with the empirical finding of a backward bending labor supply curve?*

(a) *For the typical worker, leisure is an inferior good when wages are low and a normal good when wages are high.*

Answer: As wages increase, the substitution effect tells us that workers should work more (because taking leisure has become relatively more expensive). If leisure is an inferior good, the wealth effect also tells us that workers should work more when the wage increases. Thus, *if leisure is an inferior good*, the labor supply curve *must* slope up. If leisure is a normal good, however, the wealth effect tells us that an increase in wages should cause workers to work less. Thus, when leisure is normal, substitution and wealth effects go in the opposite direction — implying that the labor supply curve can slope up or down. Either is consistent with leisure being normal, but only an upward slope is consistent with leisure being inferior.

A backward bending labor supply curve is a labor supply curve that slopes up when wages are low and down when wages are high. If leisure is inferior when wages are low (as specified in this part of the question), this is consistent with an upward slope when wages are low. If leisure is normal when wages get high, this is consistent with either an upward or a downward slope when wages are high — and it is therefore consistent with the downward slope of the backward bending labor supply curve. Thus, the statement in this part of the question is not inconsistent with the backward bending labor supply curve.

(b) *For the typical worker, leisure is a normal good when wages are low and an inferior good when wages are high.*

Answer: (Based on the first paragraph of the answer to (a)), leisure being normal when wages are low is consistent with an upward slope of labor supply when wages are low. Leisure being inferior when wages are high, however, is inconsistent with the downward slope of the backward bending labor supply curve when wages are high. So this statement is not consistent with the backward bending labor supply behavior hypothesized by labor economists.

(c) *For the typical worker, leisure is always a normal good.*

Answer (Based on the first paragraph of the answer to (a)), leisure being a normal good is consistent with both upward and downward sloping labor supply curves. Thus, if leisure is always a normal good, it could indeed be that the labor supply curve is upward sloping for low wages and downward sloping for high wages. Thus, the statement is not inconsistent with the hypothesized backward bending labor supply curve.

(d) *For the typical worker, leisure is always an inferior good.*

Answer: The labor supply curve has to be upward sloping if leisure is inferior — but the backward bending labor supply curve hypothesizes a downward slope for high wages. Thus, leisure being always inferior is not consistent with a backward bending labor supply curve.

B: *Suppose that tastes over consumption and leisure are described by a constant elasticity of substitution utility function* $u(c,\ell) = (0.5c^{-\rho} + 0.5\ell^{-\rho})^{-1/\rho}$.

(a) *Derive the labor supply curve assuming a leisure endowment L.*

Answer: From the utility maximization problem, the *leisure demand* function is

$$\ell = \frac{L}{w^{-\rho/(\rho+1)}+1}, \tag{9.30}$$

and the *labor supply function* $l(w)$ is then simply the leisure demand subtracted from the leisure endowment L; i.e.

$$l(w) = L - \frac{L}{w^{-\rho/(\rho+1)}+1} = \frac{w^{-\rho/(\rho+1)}L}{w^{-\rho/(\rho+1)}+1}. \tag{9.31}$$

(b) *Illustrate for which values of* ρ *this curve is upward sloping and for which it is downward sloping.*

Answer: It is algebraically a little easier to show how the sign of the leisure demand curve (as opposed to the labor supply curve) depends on ρ — and since the labor supply curve just has the opposite slope, we can answer the question this way. The derivative of the leisure demand curve with respect to w then is

$$\frac{\partial \ell}{\partial w} = \frac{Lw^{-(2\rho+1)/(\rho+1)}}{\left(w^{-\rho/(\rho+1)}+1\right)^2}\left[\frac{\rho}{\rho+1}\right]. \tag{9.32}$$

The non-bracketed term is unambiguously positive — which means that the equation is positive if and only if $\rho > 0$ and negative if and only if $-1 < \rho < 0$. Thus, the leisure demand curve slopes up for positive ρ and down for negative ρ. The opposite must then be true for labor supply.

You can show this also directly with the labor supply function by taking its derivative with respect to w. After a little algebraic manipulation, you can get

$$\frac{\partial l(w)}{\partial w} = -\left[\frac{\rho}{\rho+1}\right]\left(\frac{w^{-(2\rho+1)/(\rho+1)}L}{\left(w^{-\rho/(\rho+1)}+1\right)}\right)\left(1-\frac{w^{-\rho/(\rho+1)}}{\left(w^{-\rho/(\rho+1)}+1\right)^2}\right). \tag{9.33}$$

Again, all terms except for the bracketed term are positive.[1] Since there is a negative sign at the beginning of the right hand side of the equation, we can then conclude that the derivative is positive if and only if $-1 < \rho < 0$ and negative if and only if $\rho > 0$.

This should make intuitive sense: Substitution effects cause labor to increase with wages while wealth effects cause the opposite. Thus, the larger the substitution effect — i.e. the greater the substitutability between leisure and consumption — the more likely it is that the labor supply curve is upward sloping. And the elasticity of substitution between leisure and consumption increases as ρ falls. For this reason, the labor supply curve slopes up (and the leisure demand curve slopes down) if and only if ρ is below 0.

(c) *Is it possible for the backward bending labor supply curve to emerge from tastes captured by a CES utility function?*

Answer: No, it is not possible for a backward bending labor supply curve to emerge from any one CES utility function. Each such function has a fixed ρ — and, depending on what ρ is, the entire labor supply curve is either upward or downward sloping (or perfectly vertical in the case of $\rho = 0$.)

(d) *For practical purposes, we typically only have to worry about modeling tastes accurately at the margin — i.e. around the current bundles that consumers/workers are consuming. This is because low wage workers, for instance, may experience some increases in wages but not so much that they are suddenly high wage workers, and vice versa. If you were modeling worker behavior for a group of workers and you modeled each worker's tastes as CES over leisure and consumption, how would you assume* ρ *differs for low wage and high wage workers (assuming you are persuaded of the empirical validity of the backward bending labor supply curve)?*

Answer: We know from what we have done above that the labor supply curve is upward sloping for high elasticities of substitution (i.e. $-1 < \rho < 0$) and downward sloping for low

[1]The last term in parentheses is positive because the denominator in the fraction is larger than the numerator.

elasticities of substitution (i.e. $\rho > 0$). If we believe in backward bending labor supply curves but we only need to worry about behavior at the margin, we could therefore model low wage workers (for whom labor supply is upward sloping on the margin) with low values of ρ and high wage workers (for whom labor supply is downward sloping at the margin) with high values of ρ.

Exercise 9.9: *Demand for Charities and Tax Deductibility*

Policy Application: *Demand for Charities and Tax Deductibility: One of the ways in which government policy supports a variety of activities in the economy is to make contributions to those activities tax deductible. For instance, suppose you pay a marginal income tax rate t and that a fraction δ of your contributions to charity are tax deductible. Then if you give \$1 to a charity, you do not have to pay income tax on \$$\delta$ and thus you end up paying \$$\delta t$ less in taxes. Giving \$1 to charity therefore does not cost you \$1 — it only costs you \$$(1-\delta t)$.*

A: *In the remainder of the problem, we will refer to $\delta = 0$ as no deductibility and $\delta = 1$ as full deductibility. Assume throughout that giving to charity is a normal good.*

(a) *How much does it cost you to give \$1 to charity under no deductibility? How much does it cost under full deductibility?*

Answer: Under no deductibility, it costs you \$1 to give \$1. Under full deductibility, it costs you \$$(1-t)$ to give \$1 — because by giving \$1 to charity, you save \$t in taxes.

(b) *On a graph with "dollars given to charity" on the horizontal and "dollars spent on other consumption" on the vertical, illustrate a taxpayer's budget constraint (assuming the taxpayer pays a tax rate t on all income) under no deductibility and under full deductibility.*

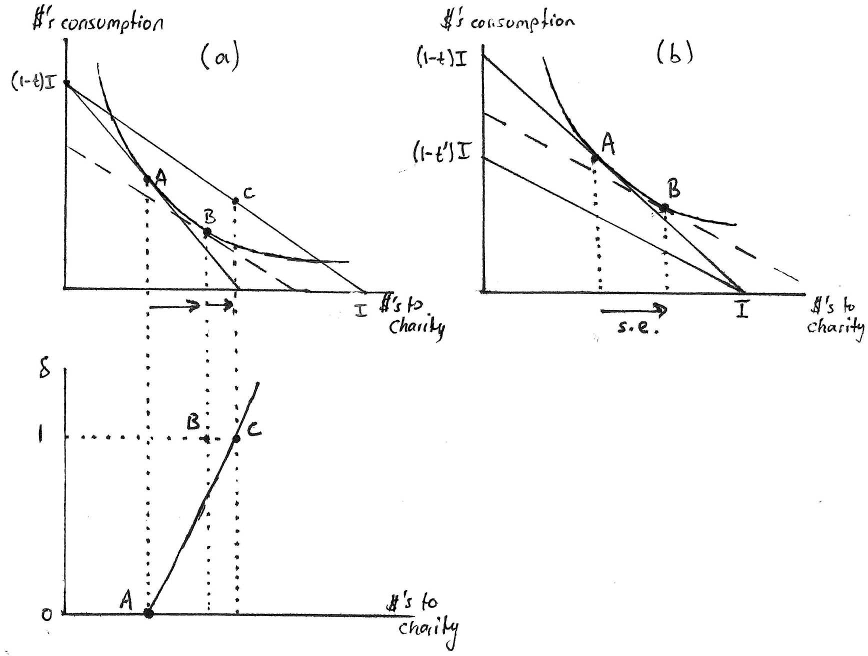
Answer: This is illustrated in panel (a) of Graph 9.15 (next page) where the steeper solid line is the no-deductibility budget and the shallower solid line is the full-deductibility budget. If no money is given to charity, then the consumer will be able to spend $(1-t)I$ — her after-tax income — on consumption. If, on the other hand, she gives all her income to charity under full deductibility, she has to pay no taxes — and is thus able to contribute her before tax income I.

(c) *On a separate graph, derive the relationship between δ (ranging from zero to 1 on the vertical) and charitable giving (on the horizontal).*

Answer: This is derived in the lower graph of panel (a) from the upper graph. Under no deductibility, the consumer optimizes at A. The substitution effect from the lower price for giving to charity under deductibility implies an increase in charitable giving to B — and the remaining income effect increases this further to C (given that we have assumed charitable giving is a normal good). Thus, as deductibility increases, charitably giving unambiguously increases.

(d) *Next, suppose that charitable giving is fully deductible and illustrate how the consumer's budget changes as t increases. Can you tell whether charitable giving increases or decreases as the tax rate rises?*

Answer: The change in the budget is illustrated in panel (b) of the graph. Under full deductibility, the maximum amount that a consumer can give to charity if she gives all her income remains the same as her tax rate changes — because if she gives her entire income, she owes no taxes under full deductibility. However, as t increases, she will not be able to consume as much in other consumption. The budget constraint therefore becomes shallower as t increases from t to t' — with the horizontal intercept remaining unchanged. Beginning at the lower tax rate t, the consumer optimizes at A. An increase in t makes giving to charity relatively cheaper — resulting in a substitution effect to B that implies greater charitable giving. However, there is an additional income effect — and, if charitable giving is a normal good, this effect will point in the opposite direction. Depending on which of these effects is bigger, a consumer might end up increasing or decreasing her charitable giving as her tax rate increases — the more substitutable charitable giving and personal consumption are, the more likely she is to increase her charitable giving as her tax rate increases.



Graph 9.15: Tax deductibility of Charitable Contributions

(e) *Suppose that an empirical economist reports the following finding: "Increasing tax deductibility raises charitable giving, and charitable giving under full deductibility remains unchanged as the tax rate changes." Can such behavior emerge from a rationally optimizing individual?*

Answer: Yes, we have shown that it can in the answers above.

(f) *Shortly after assuming office, President Barack Obama proposed repealing the Bush tax cuts — thus raising the top income tax rate to 39.6%. At the same time, he made the controversial proposal to only allow deductions for charitable giving as if the marginal tax rate were 28%. For someone who pays the top marginal income tax under the Obama proposal, what does the proposal imply for δ? What about for someone paying a marginal tax rate of 33% or someone paying a marginal tax rate of 28%?*

Answer: The Obama proposal implies that anyone whose marginal income tax rate exceeds 28% will face a cost of 72 cents for every dollar he gives to charity; i.e. $(1-\delta t) = 0.72$. For someone who pays the top marginal tax rate, we then plug $t = 0.396$ into $(1-\delta t) = 0.72$ and solve for δ to get $\delta \approx 0.71$. When the tax rates are 33% or 28%, repeating this for $t = 0.33$ and $t = 0.28$ gives us $\delta \approx 0.85$ and $\delta = 1$. The Obama proposal therefore effectively lowers the fraction δ of charitable contributions that can be deducted by high income taxpayers.

(g) *Would you predict that the Obama proposal would reduce charitable giving?*

Answer: In part (c) we showed that as deductibility δ increases, we get unambiguously more charitable giving. By the same logic — i.e. both income and substitution effects pointing in the same direction, we conclude that charitable giving will fall as deductibility δ falls. We therefore expect the Obama proposal to result in reduced charitable giving.

(h) *Defenders of the Obama proposal point out the following: After President Ronald Reagan's 1986 Tax Reform, the top marginal income tax rate was 28% — implying that it would cost* *high earners 72 cents for every dollar they contribute to charity, just as it would under the Obama proposal. If that was good enough under Reagan, it should be good enough now. In what sense it the comparison right, and in what sense is it misleading?*

Answer: The first statement is absolutely correct: For high income individuals, the cost of giving $1 to charities is 72 cents under the 1986 tax reform as well as under the Obama proposal. Put differently, both proposals set the same opportunity cost for giving to charities (for high income earners) — and thus the substitution effect is the same. The difference is the income effect because the tax rates are higher under the Obama proposal than under the Reagan reform. And the income effect would predict lower charitable giving under the Obama proposal than under the terms of the 1986 tax reform.

B: *Now suppose that a taxpayer has Cobb-Douglas tastes over charitable giving (x_1) and other consumption (x_2).*

(a) *Derive the taxpayer's demand for charitable giving as a function of income I, the degree of tax deductibility δ and the tax rate t.*

Answer: Solving the problem

$$\max_{x_1,x_2} x_1^{\alpha} x_2^{(1-\alpha)} \text{ subject to } (1-\delta t)x_1 + x_2 = (1-t)I, \tag{9.34}$$

we get

$$x_1 = \frac{\alpha(1-t)I}{(1-\delta t)}. \tag{9.35}$$

(b) *Is this taxpayer's behavior consistent with the empirical finding by the economist in part A(e) of the question?*

Answer: Yes, it is. The first part of the empirical finding said that increasing tax deductibility will increase the consumer's charitable giving. The derivative of x_1 with respect to δ is indeed positive — thus, as δ increases (i.e. as deductibility increases), x_1 increases. The second part of the empirical finding is that, under full deductibility (i.e. when $\delta = 1$), a change in the tax rate has no effect on charitable giving. Setting δ equal to 1 in equation (9.35), we get

$$x_1 = \frac{\alpha(1-t)I}{(1-t)} = \alpha I. \tag{9.36}$$

Thus, under full deductibility, charitable giving is immune to the tax rate — because the income and substitution effects exactly offset each other.

Conclusion: Potentially Helpful Reminders

1. Although we do not use the B bundle that emerges from the compensated budget in this chapter, it is useful to keep in mind where it is even as we connect only A and C to derive our demand and supply relationships. This allows us to really see the role of income and wealth effects which will continue to play important roles in later chapters.

2. The only way any of our demand curves from this chapter ever pass through our point B is if the good we are modeling is quasilinear (implying no income or wealth effects for that good). The same is true for labor (or capital) supply curves if leisure (or present consumption) is quasilinear.

3. Keep in mind that the curves we have derived will usually shift when the economic variables that are held fixed along the curves are changed. For instance, an own-price demand curve will shift to the right when income rises if the underlying good is a normal good (and to the left if it is an inferior good).

4. Own-price demand curves do not shift with changes in income if the underlying good is quasilinear. (See within-chapter exercises 9A.3 through 9A.5.)

5. If you are covering the mathematical B-part of the text, you should be able to relate the concept of demand (and supply) *curves* to demand (and supply) *functions*. Two things to keep in mind: First, the *functions* typically contain multiple variables (like income and more than one price), but the *curves* only allow one of these to vary, thus holding all others implicitly fixed. It is in that sense that we say that the *curves* are slices of the *functions*. Second, the curves have price on the vertical axis and quantity on the horizontal — but the functions are in the form $x(p)$ — i.e. the slices of the functions that hold all but one variable fixed have prices on the horizontal and quantity on the vertical. As a result, we say that the *curves* are not just slices of the functions — they are *inverse slices*.